Non-minimal coupling in inflation and inflating with the Higgs boson

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Outline

1. Inflation—"standard" approach
   - Cosmological requirements
   - Large field chaotic inflation

2. Non-minimal coupling in $\lambda \phi^4$
   - The action
   - Conformal transformation
   - Large non-minimal coupling limit
   - Generic non-minimal coupling case
   - WMAP-5 allowed parameters

3. SM Higgs as the inflaton
   - Non-minimally coupled Standard Model
   - Radiative corrections—not (too) dangerous
   - Higgs mass

4. Conclusions
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Cosmological implications

Problems in cosmology

- Flatness problem (at $T \sim M_P$ density was tuned $|\Omega - 1| \lesssim 10^{-59}$)
- Entropy of the Universe $S \sim 10^{87}$
- Size of the Universe (at $T \sim M_P$ size was $10^{29} M_P^{-1}$)
- Horizon problem

Solution

Inflation!
Cosmological implications

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Solution

Inflation!
CMB

Temperature fluctuations

Polarization

CMB spectrum

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**λφ⁴ inflation**

One scalar field

\[
S = \int d^4x \left[ \frac{\partial \mu \phi \partial^\mu \phi}{2} - V(\phi) \right], \quad V(\phi) = \frac{\lambda}{4} \phi^4
\]

Predicts primordial perturbation parameters

- COBE normalization
  \[ U/\epsilon = (0.027M_P)^4 \]
  \[ \Rightarrow \lambda \approx 10^{-13} \]

- Spectral index \( n_s = 0.95 \)
- Tensor/scalar ratio \( r = 0.26 \)
\( \lambda \phi^4 \) inflation predictions

Usual conclusion

\( \lambda \phi^4 \) is disfavoured

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Possible operators in the model+gravity

- Dimension $\leq 4$
- No new degrees of freedom (no higher derivatives)

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} R + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - V(\phi) \right.$$ 
$$- \frac{\xi}{2} \phi^2 R \right.$$ 
$$+ aR^2 + bR_{\mu\nu} R^{\mu\nu} + cR_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} + d\Box R \left. \right]$$

- The non-minimally coupled term is in fact required by the renormalization properties of the theory in curved space-time background
Non-minimal coupling in $\lambda \phi^4$

The action

Possible operators in the model+gravity

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$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} R + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - V(\phi) - \frac{\xi}{2} \phi^2 R + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + d\Box R \right]$$

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The non-minimally coupled term is in fact *required* by the renormalization properties of the theory in curved space-time background.
Non-minimally coupled scalar field— inflation

Quite an old idea

Add $\phi^2 R$ term to/instead of the usual $M_P R$ term in the gravitational action

- A.Zee’78, L.Smolin’79, B.Spokoiny’84
- D.Salopek J.Bond J.Bardeen’89

“Jordan frame” action

$$S_J = \int d^4x \sqrt{-g} \left\{ - \frac{M^2}{2} + \xi \phi^2 R + g_{\mu\nu} \frac{\partial^\mu \phi \partial^\nu \phi}{2} - \frac{\lambda}{4} \phi^4 \right\}$$
Non-minimal coupling in $\lambda \phi^4$

Conformal transformation

It is possible to get rid of the non-minimal coupling by the conformal transformation (field redefinition)

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{\xi \phi^2}{M_P^2}$$

and also redefinition of the scalar field to make canonical kinetic term

$$\frac{d\hat{\phi}}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2 \phi^2 / M_P^2}{\Omega^4}} \quad \Rightarrow \quad \begin{cases} 
\phi \simeq \hat{\phi} & \text{for } \phi < M_P / \xi \\
1 + \frac{\xi \phi^2}{M_P^2} \simeq \exp \left( \frac{2\hat{\phi}}{\sqrt{6} M_P} \right) & \text{for } \phi > M_P / \xi
\end{cases}$$

Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{\hat{g}} \left\{ - \frac{M_P^2}{2} \hat{R} + \hat{g}_{\mu\nu} \frac{\partial^\mu \hat{\phi} \partial^\nu \hat{\phi}}{2} - \frac{1}{\Omega(\hat{\phi})^4} \frac{\lambda}{4} \phi(\hat{\phi})^4 \right\}$$
Case of large $\xi$

- Easy to analyse and is in fact the main case we will need for inflation in the Standard Model
- Generic $\xi$ just interpolates between usual (minimal coupling) case and large $\xi$ case.
Inflationary potential

For $\hat{\phi} \gtrsim M_P$ :

$$U(\hat{\phi}) \simeq \frac{\lambda M_P^4}{4\xi^2} \left( 1 - \exp\left( -\frac{2\hat{\phi}}{\sqrt{6}M_P} \right) \right)^2$$
Slow roll stage

\[ \varepsilon = \frac{M_P^2}{2} \left( \frac{dU}{d\phi} \right)^2 \approx \frac{4M_P^4}{3\xi^2\phi^4} \approx \frac{4}{3}e^{-\frac{4\phi}{\sqrt{6}M_P}} \]

\[ \eta = M_P^2 \frac{d^2U}{d\phi^2} \approx \frac{4M_P^4}{3\xi^2\phi^4} \left( 1 - \frac{\xi\phi^2}{M_P^2} \right) \approx \frac{4}{3}e^{-\frac{4\phi}{\sqrt{6}M_P}} \left( 1 - e^{\frac{2\phi}{\sqrt{6}M_P}} \right) \]

Slow roll ends at \( \hat{\phi}_{\text{end}} \approx M_P \) (or \( \phi_{\text{end}} \approx M_P/\sqrt{\xi} \))

Number of e-folds of inflation at the moment \( \phi_N \) is \( N \approx \frac{6}{8} \frac{\phi_N^2 - \phi_{\text{end}}^2}{M_P^2/\xi} \)

\[ \hat{\phi}_{60} \approx 5M_P \]

COBE normalization \( U/\varepsilon = (0.027M_P)^4 \) gives

\[ \xi \approx \sqrt{\frac{\lambda}{3}} \frac{N_{\text{COBE}}}{0.027^2} \approx 49000\sqrt{\lambda} \]

Smallness of \( \lambda \) can be compensated by large \( \xi \)
CMB parameters—spectrum and tensor modes

\[ n = 1 - 6\varepsilon + 2\eta \simeq 1 - \frac{8(4N + 9)}{(4N + 3)^2} \simeq 0.97 \]

\[ r = 16\varepsilon \simeq \frac{192}{(4N + 3)^2} \simeq 0.0033 \]
Before moving on to using the Higgs field as the inflaton, let us elaborate a bit on generic $\xi$ case

What minimal $\xi$ is needed to reconcile $\lambda \phi^4$ inflation with CMB data?

S. Tsujikawa B. Gumjudpai’04
\( \xi \) dependence of \( \lambda \)

\[
\xi = 49000 \sqrt{\lambda}
\]

Graph showing the relationship between \( \log(\xi) \) and \( \lambda(\xi) \) with the equation \( \xi = 49000 \sqrt{\lambda} \).
With non-minimal coupling it is very natural for $\lambda \phi^4$ inflation to be compatible with observations!
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   - Non-minimally coupled Standard Model
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4 Conclusions
Non-minimally coupled Higgs boson

\[ S = \int d^4x \sqrt{-g} \left[ \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \frac{|D_\mu H|^2}{2} - V(H) + \bar{\psi} \not{\partial} \psi + YH\bar{\psi}_L \psi_R \right. \]

\[ \left. - \frac{M_P^2}{2} R - \xi H^\dagger H R \right] \]

COBE normalization \( U/\epsilon = (0.027M_P)^4 \) now determines \( \xi \)

\[ \xi \approx \sqrt{\frac{\lambda}{3}} \frac{N_{\text{COBE}}}{0.027^2} \approx 49000 \sqrt{\lambda} = 49000 \frac{m_H}{\sqrt{2}v} \]

Connection of the parameter \( \xi \) and the Higgs mass!

Note: \( \xi v^2 \ll M_P^2 \), so all inflationary analysis can be made just with quartic potential
After inflation—back to the SM

\[ \frac{M_P}{\xi} < \hat{\phi} < M_P : \quad U \sim \frac{\lambda M_P^2}{6 \xi^2} \hat{\phi}^2, \quad \Omega \approx 1, \quad \hat{\phi} \approx \sqrt{\frac{3}{2}} \frac{\xi h^2}{M_P}, \quad T_{\text{reh}} \gtrsim 10^{13} \text{GeV} \]

For \( \hat{\phi} \lesssim M_P / \xi \): the Standard Model
SM Higgs as the inflaton
Non-minimally coupled Standard Model

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Not the end of the story — see next talk
Radiative corrections

Could be a problem

In the ordinary situation effective potential is generated

$$\Delta U(\phi) \sim \frac{m^4(\phi)}{64\pi^2} \log \frac{m^2(\phi)}{\mu^2} + A\Lambda^2 + B\Lambda^4$$

We suppose that quadratic divergences are dealt with (eg. in dimensional regularization)
Radiative corrections

Could be a problem

In the ordinary situation effective potential is generated

$$\Delta U(\phi) \sim \frac{m^4(\phi)}{64\pi^2} \log \frac{m^2(\phi)}{\mu^2}$$

standard Yukawa interaction $m = y \cdot h$

$$\Delta U \propto -y^4 \phi^4 \log \frac{\phi^2}{\mu^2}$$

Spoils flatness of the potential (for top quark $y \sim 1$ !)
Radiative corrections

This is also cured by non-minimal coupling!

Effective potential is still generated

\[ \Delta U(\hat{\phi}) \sim \frac{m^4(\hat{\phi})}{64\pi^2} \log \frac{m^2(\hat{\phi})}{\mu^2} \]

Conformal transformation: fermions

\[ S_J = \int d^4x \sqrt{-g} \left\{ \bar{\psi} \hat{\partial} \psi + y\phi \bar{\psi} \psi \right\} \]

\[ \hat{\psi} = \Omega^{-3/2} \psi \]

\[ S_E = \int d^4x \sqrt{-\hat{g}} \left\{ \bar{\hat{\psi}} \hat{\partial} \hat{\psi} + y \frac{\phi(\hat{\phi})}{\Omega(\hat{\phi})} \bar{\hat{\psi}} \hat{\psi} \right\} \]
Radiative corrections

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\[ \Delta U(\hat{\phi}) \sim \frac{m^4(\hat{\phi})}{64\pi^2} \log \frac{m^2(\hat{\phi})}{\mu^2} \]

The interactions are suppressed now!

\[ m(\hat{\phi}) = y \frac{\phi(\hat{\phi})}{\Omega(\hat{\phi})} \xrightarrow{\hat{\phi} \to \infty} \text{const} \]

(where \( \Omega(\hat{\phi}) \propto \phi(\hat{\phi}) \) for large \( \hat{\phi} \))

\[ \Rightarrow \quad \Delta U(\hat{\phi}) \to y^4 \frac{M_P^4}{\xi^2} \left(1 - e^{-\frac{2\hat{\phi}}{\sqrt{6}M_P}}\right)^2 \log \left(\frac{m^2(\hat{\phi})}{\mu^2}\right) \to \text{const} \]
Radiative corrections

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\[ \Delta U(\hat{\phi}) \sim \frac{m^4(\hat{\phi})}{64\pi^2} \log \frac{m^2(\hat{\phi})}{\mu^2} \]

The same for self interactions

\[ m^2(\hat{\phi}) = U''(\hat{\phi}) = \frac{\lambda M_P^2}{3\xi^2} \left( 2e^{-\frac{2\hat{\phi}}{\sqrt{6}M_P}} - 1 \right) e^{-\frac{2\hat{\phi}}{\sqrt{6}M_P}} \hat{\phi} \rightarrow \infty \]

\[ \Rightarrow \quad \Delta U(\hat{\phi}) \rightarrow 0 \]
SM Higgs as the inflaton

Expected window for the Higgs mass

Standard Model should remain applicable up to

\[ \frac{M_P}{\xi} \simeq 10^{14} \text{GeV} \]

We expect the Higgs mass

\[ 130 \text{ GeV} < M_H < 190 \text{ GeV} \]
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Main conclusion

Non-minimal gravity coupling in inflationary models changes predictions a lot and in a very interesting way!

- Adding non-minimal coupling $\frac{\xi \phi^2}{2} R$ with small $\xi > 10^{-3}$ makes $\lambda \phi^4$ chaotic inflation agree with WMAP data.
- These type of models generally gives a very small amount of tensor perturbations after inflation.
- Adding non-minimal coupling $\xi H^\dagger H R$ of the Higgs field to the gravity makes inflation possible without introduction of new fields.
  - The new parameter of the model, non-minimal coupling $\xi$, relates the normalization of CMB fluctuations and the Higgs mass $\xi \simeq 49000 m_H/\sqrt{2} v$.
  - Spectral index $n_s \simeq 0.97$.
  - Tensor/scalar ratio $r \simeq 0.0033$. 
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