

PARTICLE-HOLE OPTICAL MODEL FOR GIANT-RESONANCE STRENGTH FUNCTIONS

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Abstract

An attempt is undertaken to formulate a particle-hole optical model for description of giant-resonance strength functions at arbitrary (but high enough) excitation energies. The model is based on the Bethe–Goldstone equation for the particle-hole Green function. This equation involves a specific energy-dependent particle-hole interaction that is due to virtual excitation of manyquasiparticle configurations. After energy averaging, this interaction involves an imaginary part. The analogy between the single-quasiparticle and particle-hole optical models is outlined.

1 Introduction

Damping of giant resonances (GRs) is a long-standing problem for theoretical studies. There are three main modes of GR relaxation: (i) particle-hole (p–h) strength distribution (Landau damping), which is a result of the shell structure of nuclei; (ii) coupling of (p–h)-type states with the single-particle (s.p.) continuum, which leads to direct nucleon decay of GRs; and, (iii) coupling of (p–h)-type states with manyquasiparticle configurations, which leads to the spreading effect. An interplay of these relaxation modes takes place in the GR phenomenon. A description of the giant-resonance strength function with exact allowance for the Landau damping and s.p. continuum can be obtained within the continuum-RPA (cRPA), provided the nuclear mean field and p–h interaction are fixed [1]. As for the spreading effect, it is described within microscopic and phenomenological approaches. The coupling of the (p–h)-type states, which are the doorway states (DWS) for the spreading effect, with a limited number of 2p–2h configurations is explicitly taken into account within the microscopic approaches (see, e.g., Refs. [2,3]). Some questions to the basic points of the approach could be brought up:

(1) “Thermalization” of the DWS, which form a given GR, i.e. the DWS coupling with manyquasiparticle states (MQPS) (the latter are complicated superpositions of 2p–2h, 3p–3h, . . . configurations) is not taken into account. As a result, each DWS may interact with others via 2p–2h configurations. Due to complexity of MQPS one can reasonably expect that after energy averaging the interaction of different DWS via MQPS would be close to zero (the statistical assumption). (2) The use of a limited basis of 2p–2h configurations does not allow to describe correctly the GR energy shift due to the spreading effect. The full basis of these

configurations should be formally used for this purpose. (3) With the single exception of Ref. [4], there are no studies of GR direct-decay properties within the microscopic approaches.

Within the so-called semimicroscopic approach, the spreading effect is phenomenologically taken into account directly in the cRPA equations in terms of the imaginary part of an effective s.p. optical-model potential [5,6]. Within this approach, the afore-mentioned statistical assumption is supposed to be valid and used in formulation of the approach. The GR energy shift due to the spreading effect is evaluated by means of the proper dispersive relationship, and therefore, the full basis of MQPS is formally taken into account [7]. The approach is applied to description of direct-decay properties of various GRs (the references are given in Ref. [6]). In accordance with the “pole” approximation used for description of the spreading effect within the semimicroscopic approach, the latter is valid in the vicinity of the GR energy. However, for an analysis of some phenomena it is necessary to describe the low- and/or high-energy tails of various GRs. For instance, the asymmetry (relative to 90°) of the (γn) -reaction differential cross section at the energy of the isovector giant quadrupole resonance is determined, in particular, by the high-energy tail of the isovector giant dipole resonance [8]. Another example is the isospin-selfconsistent description of the IAR damping. In particular, the IAR total width is determined by the low-energy tail of the charge-exchange giant monopole resonance [9,10].

In the present work, we attempt to formulate the p–h optical model for a phenomenological description of the spreading effect on GR strength functions at arbitrary (but high enough) energies. In this attempt, our formulation is analogous to that of the s.p. optical model [11,12]. The dispersive version of this model [11] is widely used for description of various properties of single-quasiparticle excitations at relatively high energies (see, e.g., Ref. [13]). This model can be also used for description of direct particle decay of subbarrier s.p. states [14].

2 Single-particle and particle-hole Green functions

The starting point of the formulation of the s.p. optical model is the Fourier-component of the Fermi-system s.p. Green function, $G(x, x'; \varepsilon)$, taken in the coordinate representation (see, e.g., Refs. [11,12]). In analogy with this, we start formulation of the p–h optical model from the Fourier-component of the Fermi-system p–h Green function, $\mathcal{A}(x, x'; x_1, x'_1; \omega)$, also taken in the coordinate representation. Being a kind of the Fermi-system two-particle Green function (for definitions see, e.g., Ref. [15]), \mathcal{A} satisfies the following spectral expansion:

$$\mathcal{A}(x, x'; x_1, x'_1; \omega) = \sum_s \left(\frac{\rho_s^*(x', x) \rho_s(x_1, x'_1)}{\omega - \omega_s + i0} - \frac{\rho_s^*(x'_1, x_1) \rho_s(x, x')}{\omega + \omega_s - i0} \right). \quad (1)$$

Here, $\omega_s = E_s - E_0$ is the excitation energy of an exact state $|s\rangle$ of the system and $\rho_s(x, x') = \langle s | \hat{\Psi}^+(x) \hat{\Psi}(x') | 0 \rangle$ is the transition matrix density ($\hat{\Psi}^+(x)$ is the operator of particle creation at the point x). In accordance with the expansion of Eq. (1) the p–h Green function determines the strength function $S_V(\omega)$ corresponding to an external (generally, nonlocal) s.p. field $\hat{V} = \int \hat{\Psi}^+(x) V(x, x') \hat{\Psi}(x') dx dx' \equiv [\hat{\Psi}^+ V \hat{\Psi}]$:

$$S_V(\omega) = -\frac{1}{\pi} \text{Im} [V^+ \mathcal{A}(\omega) V], \quad (2)$$

where the brackets $[\dots]$ mean the proper integrations.

The free s.p. and p-h Green functions, $G_0(x, x'; \varepsilon)$ and $\mathcal{A}_0(x, x'; x_1, x'_1; \omega)$, respectively, are determined by the mean field (via the s.p. wave functions) and the occupation numbers (only nuclei without nucleon pairing are considered). Being determined by Eq. (1), the free transition matrix densities $\rho_s^{(0)}(x, x')$ are orthogonal: $[\rho_s^{(0)*} \rho_{s'}^{(0)}] = \delta_{ss'}$. As applied to transition densities $\rho_s^{(0)}(x = x')$, this statement is false. The RPA p-h Green function, $\mathcal{A}_{RPA}(x, x'; x_1, x'_1; \omega)$, is determined also by a p-h (local) interaction $\mathcal{F}(x, x'; x_1, x'_1) = F(x, x_1)\delta(x - x')\delta(x_1 - x'_1)$, which is responsible for long-range correlations leading to formation of GRs. In particular, the Landau-Migdal forces $F(x, x_1) \rightarrow F(x)\delta(x - x_1)$ are used in realizations of the semimicroscopic approach of Refs. [5,6]. The RPA p-h Green function satisfies the expansion, that is similar to that of Eq. (1). In such a case, the RPA states $|d\rangle$ are the DWS for the spreading effect. The local RPA p-h Green function $A(x, x_1; \omega) = \mathcal{A}(x = x'; x_1 = x'_1; \omega)$ determined by the p-h interaction $F(x, x_1)$ is used for cRPA-based description of the GR strength function corresponding to a local external field $\widehat{V} = \int \widehat{\Psi}^+(x)V(x)\widehat{\Psi}(x)dx$ [1].

The s.p. and p-h Green functions satisfy, respectively, the Dyson and Bethe-Goldstone integral equations:

$$G(\varepsilon) = G_0(\varepsilon) + [G_0(\varepsilon)\Sigma(\varepsilon)G(\varepsilon)], \quad (3)$$

$$\mathcal{A}(\omega) = \mathcal{A}_{RPA}(\omega) + [\mathcal{A}_{RPA}(\omega)\mathcal{P}(\omega)\mathcal{A}(\omega)]. \quad (4)$$

The self-energy operator $\Sigma(x, x'; \varepsilon)$ and the polarization operator $\mathcal{P}(x, x'; x_1, x'_1; \omega)$ describe the coupling, correspondingly, of single-quasiparticle and (p-h)-type states with MQPS. Analytical properties of G and Σ are nearly the same. A similar statement can be made for \mathcal{A} and \mathcal{P} . The quantities $\Sigma(\varepsilon)$ and $\mathcal{P}(\omega)$ both exhibit a sharp energy dependence due to a high density of poles corresponding to virtual excitation of MQPS. Concluding this Section, we present the alternative equation for the p-h Green function:

$$\mathcal{A}(\omega) = \mathcal{A}_0(\omega) + [\mathcal{A}_0(\omega) (\mathcal{F} + \mathcal{P}(\omega)) \mathcal{A}(\omega)], \quad (5)$$

which follows from Eq. (4) and from the equation for \mathcal{A}_{RPA} . The latter is similar to Eq. (4) and contains the p-h interaction \mathcal{F} .

3 Transition to the optical models

Since the density of MQPS, ρ_m , is large and described by statistical formulae, only the quantities $\overline{\Sigma}(x, x'; \varepsilon)$ and $\overline{\mathcal{P}}(x, x'; x_1, x'_1; \omega)$ averaged over an interval $J \gg \rho_m^{-1}$ can be reasonably parameterized. As applied to $\overline{\Sigma}(\varepsilon) = \Sigma(\varepsilon + iJSgn(\varepsilon - \mu))$, it is done, e.g., in Refs. [11,12]:

$$\overline{\Sigma}(x, x'; \varepsilon) = Sgn(\varepsilon - \mu) \{-iw(x; \varepsilon) + p(x; \varepsilon)\} \delta(x - x'). \quad (6)$$

Here, μ is the chemical potential and $w(x; \varepsilon)$ is the imaginary part of a (local) optical-model potential. Assuming that the radial dependencies of p and w are the same, i.e. $w(x; \varepsilon) \rightarrow w(r)w(\varepsilon)$ and $p(x; \varepsilon) \rightarrow w(r)p(\varepsilon)$, the intensity of the real addition to the mean field, $p(\varepsilon)$, has been expressed in terms of $w(\varepsilon)$ via the corresponding dispersive relationship [11]. It is noteworthy, that the optical-model addition to the mean field can be taken to be local, i.e. $\overline{\Sigma}(x, x'; \varepsilon) \sim \delta(x - x')$, in view of a large momentum transfer (of order of the Fermi momentum) at the “decay” of single-quasiparticle states into MQPS. The energy averaged

s.p. Green function $\overline{G}(x, x'; \varepsilon)$ satisfies to the Eq. (3), which involves in such a case the quantity $\overline{\Sigma}$ of Eq. (6). Actually, \overline{G} is the Green function of the Schrödinger equation, which involves the addition to the mean field considered above.

The energy-averaged polarization operator can be parameterized similarly to Eq. (6):

$$\overline{P}(x, x'; x_1, x'_1; \omega) = \{-i\mathcal{W}(x, x'; \omega) + P(x, x'; \omega)\} \delta(x - x_1) \delta(x' - x'_1). \quad (7)$$

Assuming that the coordinate dependencies of the quantities P and \mathcal{W} are the same, i.e. $\mathcal{W}(x, x'; \omega) \rightarrow \mathcal{W}(x, x')\mathcal{W}(\omega)$ and $P(x, x'; \omega) \rightarrow \mathcal{W}(x, x')P(\omega)$, we can express $P(\omega)$ in terms of $\mathcal{W}(\omega)$ via the corresponding dispersive relationship. The example of such a relationship is given in Ref. [7]. In accordance with Eqs. (4), (5), (7) the energy-averaged p-h Green function satisfies to the equivalent equations:

$$\overline{\mathcal{A}}(\omega) = \mathcal{A}_{RPA}(\omega) + [\mathcal{A}_{RPA}(\omega)\overline{P}(\omega)\overline{\mathcal{A}}(\omega)], \quad (8)$$

$$\overline{\mathcal{A}}(\omega) = \mathcal{A}_0(\omega) + [\mathcal{A}_0(\omega) (\mathcal{F} + \overline{P}(\omega)) \overline{\mathcal{A}}(\omega)]. \quad (9)$$

Formally, Eqs. (7)–(9) are the basic equations of the p-h optical model. In particular, the energy-averaged strength function is determined by Eq. (2) with substitution $\mathcal{A}(\omega) \rightarrow \overline{\mathcal{A}}(\omega)$.

To realize the model in practice, a reasonable parametrization of $Im\overline{P}$ should be done with taking the statistical assumption into account. For this purpose, we consider the quantity \mathcal{A}_{RPA} within the discrete-RPA (dRPA) in the “pole” approximation. In accordance with Eq. (1), we have

$$\mathcal{A}_{RPA}(x, x'; x_1, x'_1; \omega) \rightarrow \sum_d \rho_d^*(x', x) \rho_d(x_1, x'_1) (\omega - \omega_d + i0)^{-1}. \quad (10)$$

The statistical assumption $[\rho_d^* \overline{P} \rho_{d'}] \sim \delta_{dd'}$ is fulfilled, provided that: (i) the intensity $\mathcal{W}(x, x'; \omega)$ is nearly constant within the nuclear volume, i.e. $\mathcal{W}(x, x'; \omega) \rightarrow \mathcal{W}(\omega)$; and, (ii) the dRPA transition matrix densities are orthogonal, i.e. $[\rho_d^* \rho_{d'}] = \delta_{dd'}$. Under these assumptions, the solution of Eq. (8) can be easily obtained in the pole approximation: $\overline{\mathcal{A}}(\omega) = \mathcal{A}_{RPA}(\omega - \omega_d + i\mathcal{W}(\omega) - P(\omega))$. As a result, the energy-averaged strength functions is the superimposition of the DWS resonances:

$$\overline{S}_V(\omega) = -\frac{1}{\pi} Im \sum_d |[V^+ \rho_d]|^2 (\omega - \omega_d + i\mathcal{W}(\omega) - P(\omega))^{-1}. \quad (11)$$

The quantity $2\mathcal{W}$ can be considered as the mean DWS spreading width $\langle \Gamma_d^\downarrow \rangle$, which might be larger than the mean energy interval between neighbouring DWS resonances.

A few points are noteworthy in conclusion of this Section. Formally, the p-h optical model is valid at arbitrary (but high enough) excitation energy. The low limit is determined by the possibility of using the statistical formulae to describe the MQPS density. Within the semimicroscopic approach, the substitution like $\omega \rightarrow \omega + i\mathcal{W}(\omega) - P(\omega)$ is used in the cRPA equations to take the spreading effect phenomenologically into account in the “pole” approximation together with the statistical assumption [6,7]. Within the s.p. optical model the statistical assumption for “decay” of different s.p. states with the same angular momentum and parity into MQPS seems to be valid. At high excitation energies $|\varepsilon - \mu|$, when the empirical value of $w(\varepsilon)$ is comparable with the energy interval between the aforementioned s.p. states, the empirical radial dependence $w(r)$ becomes nearly constant within the nuclear volume (see, e.g., Refs. [13]).

4 “Single-level” giant resonance

The p–h optical model can be simply realized in terms of the energy-averaged local p–h Green function $\bar{A}(x, x_1; \omega)$ to describe the strength function of a “single-level” GR, because in such a case there is no need for the statistical assumption. Being more simple, the equations like (4), (5), (7)–(9) are actually the straight-forward extension of the corresponding cRPA equations. In practice, within the cRPA it is more convenient to use the equation for the effective field $\bar{V}(x, \omega)$, which corresponds to a local external field $V(x)$ and is determined in accordance with the relationship: $[V\bar{A}(\omega)] = [\bar{V}(\omega)A_0(\omega)]$. The effective field determines the strength function:

$$\bar{S}_V(\omega) = -\frac{1}{\pi} \text{Im} [V_0 A_0(\omega) \bar{V}(\omega)], \quad (12)$$

and satisfies to the equation:

$$\bar{V}(\omega) = V + [(F + \bar{\Pi}(\omega)) A_0(\omega) \bar{V}(\omega)]. \quad (13)$$

The energy-averaged local polarization operator is parameterized similarly to Eq. (7):

$$\bar{\Pi}(x, x_1; \omega) = C \{-iW(x; \omega) + P(x; \omega)\} \delta(x - x_1). \quad (14)$$

Here, $C = 300 \text{ MeV fm}^3$ is the value often used in parametrization of the Landau–Migdal forces; W and P are the dimensionless quantities, which can be parameterized as follows: $W(x; \omega) \rightarrow W(r)W(\omega)$ and $P(x; \omega) \rightarrow W(r)P(\omega)$, where $P(\omega)$ is determined by $W(\omega)$ via the corresponding dispersive relationship [7].

Due to strong coupling with s.p. continuum the high-energy GRs (they are mostly the overtones of corresponding low-energy GRs) can be roughly considered as the “one-level” ones. Being the IAR overtone, the charge-exchange (in the β^- -channel) giant monopole resonance (GMR⁽⁻⁾) is related to these GRs. Within the isospin-selfconsistent description of the IAR damping [9,10], the low-energy “tail” of the GMR⁽⁻⁾ in the energy dependence of the “Coulomb” strength function $\bar{S}_C^{(-)}(\omega)$ determines the IAR total width Γ_A via the nonlinear equation:

$$\Gamma_A = 2\pi S_A^{-1} \bar{S}_C^{(-)}(\omega = \omega_A). \quad (15)$$

Here, $S_A \simeq (N - Z)$ is the IAR Fermi strength, ω_A is the IAR energy, and the “Coulomb” strength function corresponds to the external field $V(x) \rightarrow V_C^{(-)} = (U_C(r) - \omega_A + \frac{i}{2}\Gamma_A) \tau^{(-)}$, where $U_C(r)$ is the mean Coulomb field. Strength function $\bar{S}_C^{(-)}(\omega)$ exhibits a wide resonance corresponding to the GMR⁽⁻⁾. In Fig. 1, we present the strength function calculated for the ²⁰⁸Pb parent nucleus within: (i) the cRPA (in such a case the strength function $S_C^{(-)}(\omega = \omega_A)$ determines the IAR total escape width found without taking the isospin-forbidden spreading effect into account [16]); (ii) the semimicroscopic approach [10] and, (iii) the p–h optical model by Eqs. (12)–(14). All the model parameters, parameterization of the imaginary part of the effective s.p. optical-model potential $I(r; \omega)$ [10] and parameterization of $W(r; \omega)$ in Eq. (14) are taken the same in both approaches. The intensities of $I(r; \omega)$ and $W(r; \omega)$ are chosen to reproduce in calculations the observable total width of the GMR⁽⁻⁾ in ²⁰⁸Bi ($\simeq 15 \text{ MeV}$). Both approaches lead to the similar results, which are not exactly the same for the low-energy “tail” of the GMR⁽⁻⁾ at $\omega \simeq \omega_A$. Irregularities in the energy dependence of $\bar{S}_C^{(-)}(\omega)$ calculated within the p–h optical model are explained by the fact that the GMR⁽⁻⁾ can be roughly considered as the “one-level” one.

5 Summary and perspectives

In the present work, the particle-hole optical model for giant-resonance strength functions has been formulated in terms of the energy-averaged nonlocal particle-hole Green function. The equation for this Green function contains a specific energy-dependent particle-hole interaction, which is due to virtual excitation of manyquasiparticle configurations. The intensity of the imaginary part of this interaction should be taken nearly constant within nuclear volume to satisfy to the statistical assumption on the independent spreading of different particle-hole-type states that form a given giant resonance. The strength function of the “single-level” giant resonance can be described in terms of the energy-averaged local particle-hole Green function.

Along with numerical realizations, the particle-hole optical model can be extended to describe direct particle decays of giant resonances. These points are under consideration.

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References

- [1] S. Shlomo, G. Bertsch, Nucl. Phys. A 243, 507 (1975).
- [2] G.F. Bertsch, P.F. Bortignon, R.A. Broglia, Rev. Mod. Phys. 55, 287 (1983).
- [3] S. Kamerdziev, J. Speth, G. Tertychny, Phys. Rep. 393, 1 (2004).
- [4] G. Colò et al., Phys. Rev. C 50, 1496 (1994).
- [5] M.L. Gorelik, I.V. Safonov, M.H. Urin, Phys. Rev. C 69, 054322 (2004).
- [6] M.H. Urin, Nucl. Phys. A 811, 107 (2008).
- [7] B.A. Tulupov, M.H. Urin, Phys. At. Nucl. 72, 737 (2009).
- [8] M.L. Gorelik, B.A. Tulupov, M.H. Urin, Phys. At. Nucl. 69, 598 (2006).
- [9] I.V. Safonov, M.G. Urin, Bull. Rus. Acad. Sci. Phys. 67, 44 (2003).
- [10] M.L. Gorelik, V.S. Rykovanov, M.G. Urin, Bull. Rus. Acad. Sci. Phys. 73, N.11 (2009).
- [11] C. Mahaux, S. Sartor, Adv. Nucl. Phys. 20, 1 (1991).
- [12] M.G. Urin, “Relaxation of nuclear excitations”, Energoatomizdat, Moscow, 1991 (in Russian).
- [13] E.A. Romanovskii et al., Phys. At. Nucl. 63, 399 (2000); O.V. Besspalova et al., Phys. At. Nucl. 69, 796 (2006).
- [14] G.A. Chekomazov, M.H. Urin, Phys. Lett. B 349, 400 (1995); Phys. At. Nucl. 61, 375 (1998).

[15] A.B. Migdal, “Theory of Finite Fermi-Systems and Applications to Atomic Nuclei”, Interscience, New York, 1967.

[16] M.L. Gorelik and M.H. Urin, Phys. Rev. C 63, 064312 (2001).

Caption to Fig. 1. The “Coulomb” strength function calculated for the ^{208}Pb parent nucleus within the cRPA (thin line), the semimicroscopic approach (dash-dotted line), and p-h optical model (full line). The arrow indicates the IAR energy.

