Spin-flavor oscillations of ultrarelativistic neutrinos in an external magnetic field

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Mechanisms of neutrino oscillations

- ***Neutrino flavor oscillations***: neutrinos of one flavor are converted into neutrinos of another flavor

$$\nu_\beta \rightleftarrows \nu_\alpha$$

- ***Neutrino spin oscillations***: left-handed neutrinos are converted into right-handed neutrinos of the *same* flavor

$$\nu_{\beta \mathrm{L}} \rightleftarrows \nu_{\beta \mathrm{R}}$$

- ***Neutrino spin-flavor oscillations***: left-handed neutrinos are converted into right-handed neutrinos of *another* flavor

$$\nu_{\beta \mathrm{L}} \rightleftarrows \nu_{\alpha \mathrm{R}}$$

Basics of neutrino flavor oscillations

- There are two sets of neutrinos.
- *Flavor* neutrinos (participate in EW interactions, i.e. interact with background particles).
- *Massive* neutrinos (have *definite* masses).
- Mass eigenstates evolve in time as plane waves with *definite* energies.
- Flavor neutrinos are superposition of mass eigenstates.

Pontecorvo formula

- Neutrinos are relativistic particles ($t \approx x$ and $|\mathbf{p}| \approx E$) we get transition and survival probabilities

$$P_{\nu_\beta \to \nu_\alpha}(x) = \left| \langle \nu_\alpha | \nu_\beta(x) \rangle \right|^2 = \sin^2(2\theta) \sin^2\left(\frac{\delta m^2}{4E} x \right),$$

$$P_{\nu_\beta \to \nu_\beta}(x) = \left| \langle \nu_\beta | \nu_\beta(x) \rangle \right|^2 = 1 - P_{\nu_\beta \to \nu_\alpha}(x)$$

Difficulties of the standard quantum mechanical approach

- Do different neutrino eigenstates have equal energies of equal momenta or equal velocities?
- Do oscillations happen in time or in space?
- Is it necessary to treat neutrinos as spinor particles?
- Should one take into account the coordinate dependence of the neutrino wave function?
- How can one describe oscillations of non-relativistic neutrinos?

Evolution of first quantized flavor neutrinos

M.D., Phys. Lett. **B 610** (2005) 262

M.D., hep-ph/0609139

Effective Lagrangian and initial condition problem

- Flavor neutrinos Lagrangian (ν_α, ν_β)

$$\mathcal{L} = \sum_{\ell=\alpha\beta} \bar{\nu}_\ell (\mathrm{i}\gamma^\mu \partial_\mu - m_\ell)\nu_\ell - \sum_{\substack{\ell,\ell'=\alpha\beta \\ \alpha\neq\beta}} \Delta_{\ell\ell'} \bar{\nu}_\ell \nu_{\ell'}$$

- Initial condition $\nu_\ell(\mathbf{r}, t=0) = \xi_\ell(\mathbf{r})$

- Fields distributions at subsequent moments of time $\nu_\ell(\mathbf{r}, t) = ?$ at $t > 0$

Mass eigenstates

- Mass eigenstates are introduced in the usual way

$$\nu_\ell(\mathbf{r},t) = \sum_a U_{\ell a}\psi_a(\mathbf{r},t)$$

- We can find the fields distributions of ψ_a

$$\psi_a(\mathbf{r},t) = \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^{3/2}}[a_\zeta(\mathbf{p})u_\zeta(\mathbf{p})e^{-\mathrm{i}E_\zeta t} + b_\zeta(\mathbf{p})v_\zeta(\mathbf{p})e^{\mathrm{i}E_\zeta t}]e^{\mathrm{i}\mathbf{pr}}$$

- $a_\zeta(\mathbf{p})$ and $b_\zeta(\mathbf{p})$ are *not* the creation and annihilation operators

Evolution of mass eigenstates

- Fields distributions of massive neutrinos, depend on the Fourier transforms of the initial conditions

$$\psi_a(\mathbf{r},t) = \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \sum_{\zeta=\pm 1} \Big\{ \left(u_\zeta \otimes u_\zeta^\dagger\right) e^{-\mathrm{i}E_\zeta t}$$

$$+ \left(v_\zeta \otimes v_\zeta^\dagger\right) e^{\mathrm{i}E_\zeta t} \Big\} e^{\mathrm{i}\mathbf{pr}} \psi_a(\mathbf{p},0)$$

Evolution of flavor neutrinos

- Fields distributions of flavor neutrinos

$$\nu_\ell(\mathbf{r},t)=\sum_{a\ell'}U_{\ell a}(U^{-1})_{a\ell'}\int \mathrm{d}^3\mathbf{r}' S_a(\mathbf{r}'-\mathbf{r},t)(-\mathrm{i}\gamma^0)\xi_{\ell'}(\mathbf{r}')$$

- Pauli-Jordan function

$$S_a(\mathbf{r},t)=(\mathrm{i}\gamma^\mu\partial_\mu+m_a)D_a(\mathbf{r},t),$$

$$D_a(\mathbf{r},t)=\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}e^{\mathrm{i}\mathbf{pr}}\frac{\sin E_a t}{E_a}$$

Evolution of two mixed flavor neutrinos

- Initial conditions $\xi_\beta(\mathbf{r}) = e^{\mathrm{i}\mathbf{kr}}\xi_0,\ \xi_\alpha(\mathbf{r}) = 0$
- We obtain the field distribution of $\boldsymbol{\nu_\alpha}$ in high initial momentum approximation $(\boldsymbol{k \gg m_{1,2}})$

$$\nu_\alpha(\mathbf{r},t) = e^{\mathrm{i}\mathbf{kr}} \sin 2\theta \sin\left[\Phi(k)t\right]\left\{\sin\left[\sigma(k)t\right] + \mathrm{i}(\boldsymbol{\alpha}\mathbf{n})\cos\left[\sigma(k)t\right]\right\}\xi_0$$

$$+\mathcal{O}(m_a / k),$$

$$\Phi(k) \approx \frac{\delta m^2}{4k},\ \sigma(k) \approx k + \frac{m_1^2 + m_2^2}{4k}$$

Transition and survival probabilities

- Intensities $$I(t) = \left|\nu_\ell(\mathbf{r},t)\right|^2 = P(t)$$

- Transition and survival probabilities

$$P_{\nu_\beta \to \nu_\alpha}(t) = \sin^2(2\theta)\sin^2\left(\frac{\delta m^2}{4k}t\right) + \mathcal{O}(m_a^2/k^2),$$

$$P_{\nu_\beta \to \nu_\beta}(t) = 1 - P_{\nu_\beta \to \nu_\alpha}(t)$$

Discussion

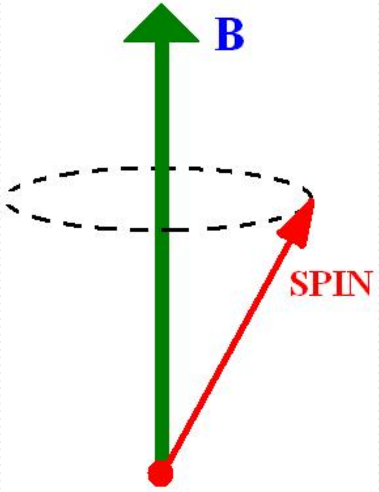
- We solved the initial condition problem for the mixed spinor fields and applied it for neutrino flavor oscillations.
- The Pontecorvo formula was re-derived. Neutrino oscillations occur in time.
- The new rapidly oscillating terms on frequency $\omega_{\mathrm{rapid}}=k+(m_1^2+m_2^2)/k$ appear in the transition probability formula. They are suppressed by the small factor $m_a/k \ll 1$.
- It was demonstrated that to obtain the stable oscillations picture one has to prepare rather broad initial wave packet (see also M.D., hep-ph/0610047).
- This approach was applied for the description of neutrino oscillations in matter in M.D., Eur. Phys. J. **C 47** (2006) 437.

Evolution of Dirac neutrinos interacting with an external magnetic field

M.D. & Jukka Maalampi,
hep-ph/0701209

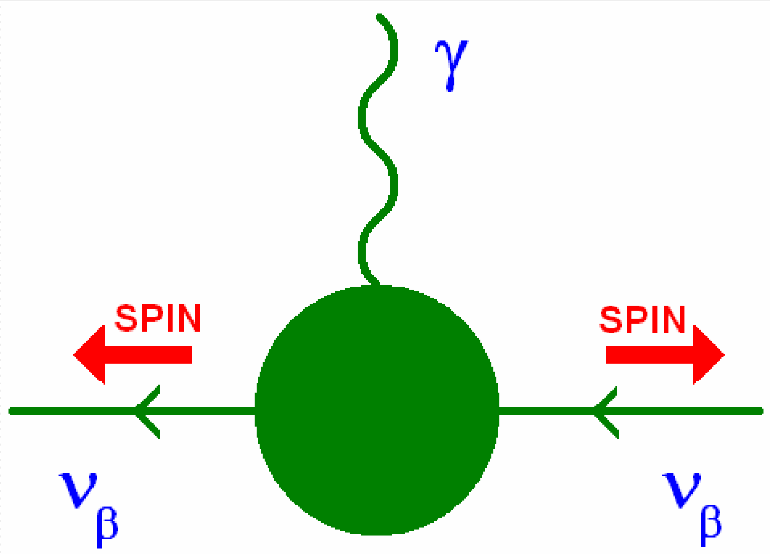
Neutrinos in an external electromagnetic field

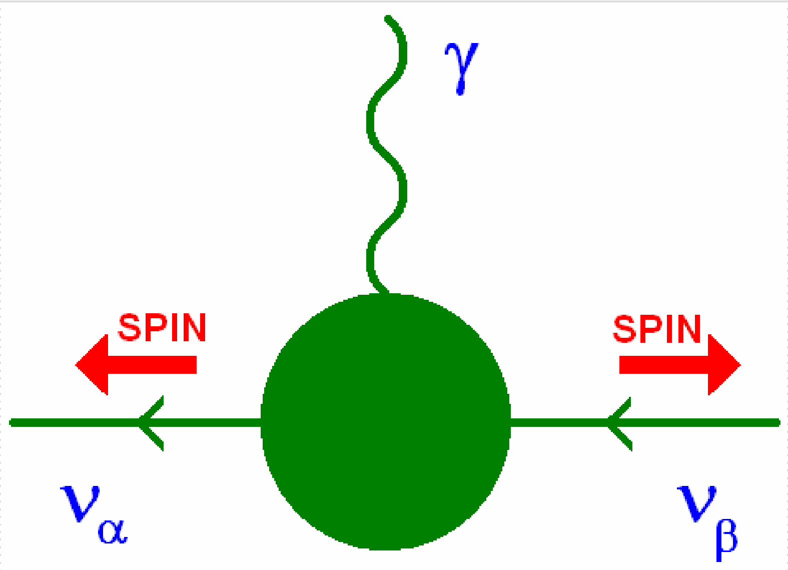
- If a neutrino is a massive particle, it can have a non-zero magnetic moment $\boldsymbol{\mu}$
- Spin of a neutrino, having the magnetic moment, rotates around the magnetic field



$$\frac{d\zeta}{dt} = 2\mu[\zeta \times \mathbf{B}]$$

Neutrino spin-flip in an electromagnetic field





$$\sim \bar{\nu}_\beta \sigma_{\mu\nu} \nu_\beta F^{\mu\nu} \qquad \sim \bar{\nu}_\alpha \sigma_{\mu\nu} \nu_\beta F^{\mu\nu}$$

Effective Lagrangian

- The Lagrangian for the Dirac neutrinos system interacting with an external magnetic field by means of the magnetic moments is

$$\mathcal{L} = \sum_{\ell=\alpha\beta} \bar{\nu}_\ell (\mathrm{i}\gamma^\mu \partial_\mu - m_\ell)\nu_\ell$$

$$- \sum_{\substack{\ell,\ell'=\alpha\beta \\ \alpha\neq\beta}} \Delta_{\ell\ell'} \bar{\nu}_\ell \nu_{\ell'} - \frac{1}{2} \sum_{\ell,\ell'=\alpha\beta} M_{\ell\ell'} \bar{\nu}_\ell \sigma_{\mu\nu} \nu_{\ell'} F^{\mu\nu}$$

Magnetic moments matrix in the mass eigenstates basis

- For Dirac neutrinos generally we have four magnetic moments in any basis

$$(\mu_{ab}) =$$

$$\begin{pmatrix} \cos^2\theta M_{\alpha\alpha} + \sin^2\theta M_{\beta\beta} + \sin 2\theta M_{\alpha\beta}; & -\sin 2\theta (M_{\alpha\alpha} - M_{\beta\beta})/2 + \cos 2\theta M_{\alpha\beta} \\ -\sin 2\theta (M_{\alpha\alpha} - M_{\beta\beta})/2 + \cos 2\theta M_{\alpha\beta}; & \cos^2\theta M_{\beta\beta} + \sin^2\theta M_{\alpha\alpha} + \sin 2\theta M_{\alpha\beta} \end{pmatrix},$$

$$\mu_{aa} = \mu_a, \ \ \mu_{ab} = \mu$$

- **Reminder!** Majorana neutrinos “effectively” have only transition magnetic moments in any basis

Dirac equation for the mass neutrino eigenstates in the external magnetic field

$$\mathrm{i}\dot{\psi}_a = \mathcal{H}_a \psi_a + \mathrm{V}\psi_b,$$

$$\mathcal{H}_a = (\boldsymbol{\alpha}\mathbf{p}) + \rho_3 m_a - \mu_a \rho_3 \Sigma_3 B,$$

$$\mathrm{V} = -\mu \rho_3 \Sigma_3 B,$$

$$\mathbf{B} = (0, 0, B), \ \ \mathbf{E} = 0$$

Special case of the magnetic moments matrix

- We suppose that magnetic moments matrix is close to diagonal. Here we assume that $\boldsymbol{\theta \approx \pi/4}$

$$\left|\left(M_{\alpha\alpha} - M_{\beta\beta}\right)/2\right| \ll \left|\left(M_{\alpha\alpha} - M_{\beta\beta}\right)/2 \pm M_{\alpha\beta}\right|$$

- The mixing between tau and muon neutrinos is close to the maximal and

$$M_{\ell\ell} \sim 10^{-19} (m_\ell / \mathrm{eV}) \mu_{\mathrm{B}}, \quad M_{\ell\ell'} < 10^{-10} \mu_{\mathrm{B}}$$

Energy levels for a neutrino in the magnetic field

$$E_a^{(\zeta)} = \sqrt{p_3^2 + \mathcal{E}_a^{(\zeta)2}}, \quad \mathcal{E}_a^{(\zeta)} = \mathcal{E}_a - \zeta\mu_a B,$$

$$\mathcal{E}_a = \sqrt{m_a^2 + p_1^2 + p_2^2}$$

- The quantum number $\zeta=\pm 1$ characterizes the spin direction with respect to the magnetic field (it is the eigenvalue of the operator $\mathbf{\Pi}_a = m_a \mathbf{\Sigma}_3 + \rho_2 [\mathbf{\Sigma}\times\mathbf{p}]_3 - \mu_a B$)

Basis spinors in the magnetic field

$$u_a^{(\zeta)} = \frac{1}{2\sqrt{E_a^{(\zeta)}}} \begin{pmatrix} \phi_a^+ \alpha_a^+ \\ -\zeta \phi_a^- \alpha_a^- e^{\mathrm{i}\varphi} \\ \phi_a^+ \alpha_a^- \\ \zeta \phi_a^- \alpha_a^+ e^{\mathrm{i}\varphi} \end{pmatrix}, \quad v_a^{(\zeta)} = \frac{1}{2\sqrt{E_a^{(\zeta)}}} \begin{pmatrix} \phi_a^+ \alpha_a^- \\ \zeta \phi_a^- \alpha_a^- e^{\mathrm{i}\varphi} \\ -\phi_a^+ \alpha_a^- \\ \zeta \phi_a^- \alpha_a^- e^{\mathrm{i}\varphi} \end{pmatrix},$$

$$\phi_a^\pm = \sqrt{1 \pm \zeta m_a / \mathcal{E}_a}, \quad \alpha_a^\pm = \sqrt{E_a^{(\zeta)} \pm \zeta \mathcal{E}_a^{(\zeta)}}, \quad \tan\varphi = p_2 / p_1$$

Initial conditions for spin-flavor oscillations

- We suppose that only left-polarized neutrinos of one flavor are presented initially

$$\nu_\beta^{\mathrm{L}}(\mathbf{r},0) = e^{i\mathbf{kr}}\xi_0, \;\; \nu_\beta^{\mathrm{R}}(\mathbf{r},0) = 0,$$

$$\nu_\alpha^{\mathrm{L}}(\mathbf{r},0) = 0, \;\; \nu_\alpha^{\mathrm{R}}(\mathbf{r},0) = 0,$$

$$\nu_{\mathrm{L,R}} = \frac{1}{2}\left\{1 \mp \frac{(\boldsymbol{\Sigma}\mathbf{k})}{k}\right\}\nu$$

Transition probability for spin-flavor oscillations

$$P_{\nu_\beta^{\mathrm{L}} \to \nu_\alpha^{\mathrm{R}}}(t) = \sin^2(2\theta)\{\sin^2(\delta\mu B t)\cos^2(\bar{\mu} B t)$$

$$+\sin(\mu_1 B t)\sin(\mu_2 B t)\sin^2[\Phi(k)t]\} + \mathcal{O}(m_a^2/k^2),$$

$$\delta\mu = (\mu_1 - \mu_2)/2,\ \ \bar{\mu} = (\mu_1 - \mu_2)/2$$

- It is also possible to take into account further corrections in $\boldsymbol{\mu}$ (see our work hep-ph/0701209)
- Transition probability for Majorana neutrinos

$$P_{\mathrm{Majorana}}(t) = \frac{(M_{\alpha\beta}B)^2}{(M_{\alpha\beta}B)^2 + (\Phi(k)\cos 2\theta)^2}\sin^2\left(\sqrt{(M_{\alpha\beta}B)^2 + (\Phi(k)\cos 2\theta)^2}\,t\right)$$

Limiting case

- If we suppose that $\mu_1 = \mu_2 = \mu_0$, we receive independent flavor and spin oscillations

$$P_{\nu_\beta^{\mathrm{L}} \to \nu_\alpha^{\mathrm{R}}}(t) = \sin^2(2\theta)\sin^2\left[\frac{\delta m^2 t}{4k}\right] \times \sin^2(\mu_0 B t)$$

$$P_{\mathrm{FLAVOR}}(t) = \sin^2(2\theta)\sin^2\left[\frac{\delta m^2 t}{4k}\right]$$

$$P_{\mathrm{SPIN}}(t) = \sin^2(\mu_0 B t)$$

Discussion

- We applied the previously elaborated method (initial condition problem) for the description of neutrinos interacting with an external electromagnetic field.
- We studied spin-flavor oscillations of Dirac neutrinos in an external magnetic field and received the new transition probability formula.
- Our approach again reveals the rapidly oscillating terms on the frequency $\omega_{\mathrm{rapid}} \approx k$ in the transition probability formula. They are suppressed by the small factor $m_a/k \ll 1$.

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