Lorentz Invariance Violation in Braneworld Models

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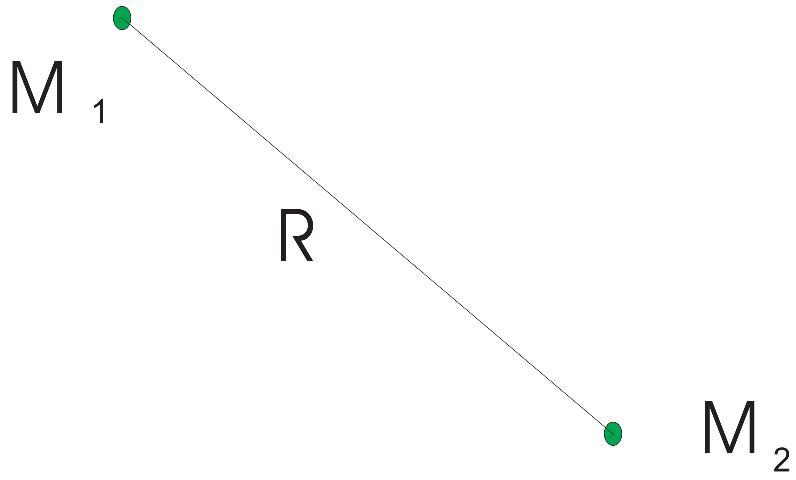
Outline

- Extra Dimensions. Introduction
- Kaluza–Klein Compactification in Field Theory and String Theory
- Large Extra Dimensions. Randall – Sundrum Model
- Violation of Lorentz Invariance
- No–Go Theorem. How to evade it?
- Spectrum of Excitations
- Open Questions

Newtonian Gravity



$$V(r) = -\frac{8\pi}{M_{pl}^2}\frac{M_1 M_2}{r}$$

flat space

$$ds^2 = dt^2 - d\mathrm{x}^2 = g_{\mu\nu} dx^\mu dx^\nu$$

This is not the case for curved background.

In GR we investigate how to find the spacetime metric tensor.

General Relativity

Einstein equation "Geometry = Physics"

$$G_{AB}(g_{AB}, \partial g_{AB}, \ldots) := R_{AB} - \frac{1}{2} R g_{AB} = T_{AB}(E, p, \ldots)$$

D(D+1)/2 PDEs

GR describes perfectly phenomena at large distances.

Gravitons

Actually gravity is not renormalizable theory but nevertheless one can consider quanta of gravity – gravitons

Excitations over solutions of Einstein eqns

$\tilde{g}_{AB} = g_{AB} + h_{AB}$

Gauge – traceless and transverse (TT) conditions

$$h_A^A = 0, \quad \partial_i h_{ij} = 0$$

Linearized equation

$$\mathscr{O} h_{AB} = \lambda h_{AB}$$

does not feel tensor structure of h_{AB}.

Extra Dimensions

Spacetime $\mathbb{R}^{1,d}$ – d+1 Minkowski space

$$ds^2 = dt^2 - d\mathrm{x}^2$$

Gravitational potential

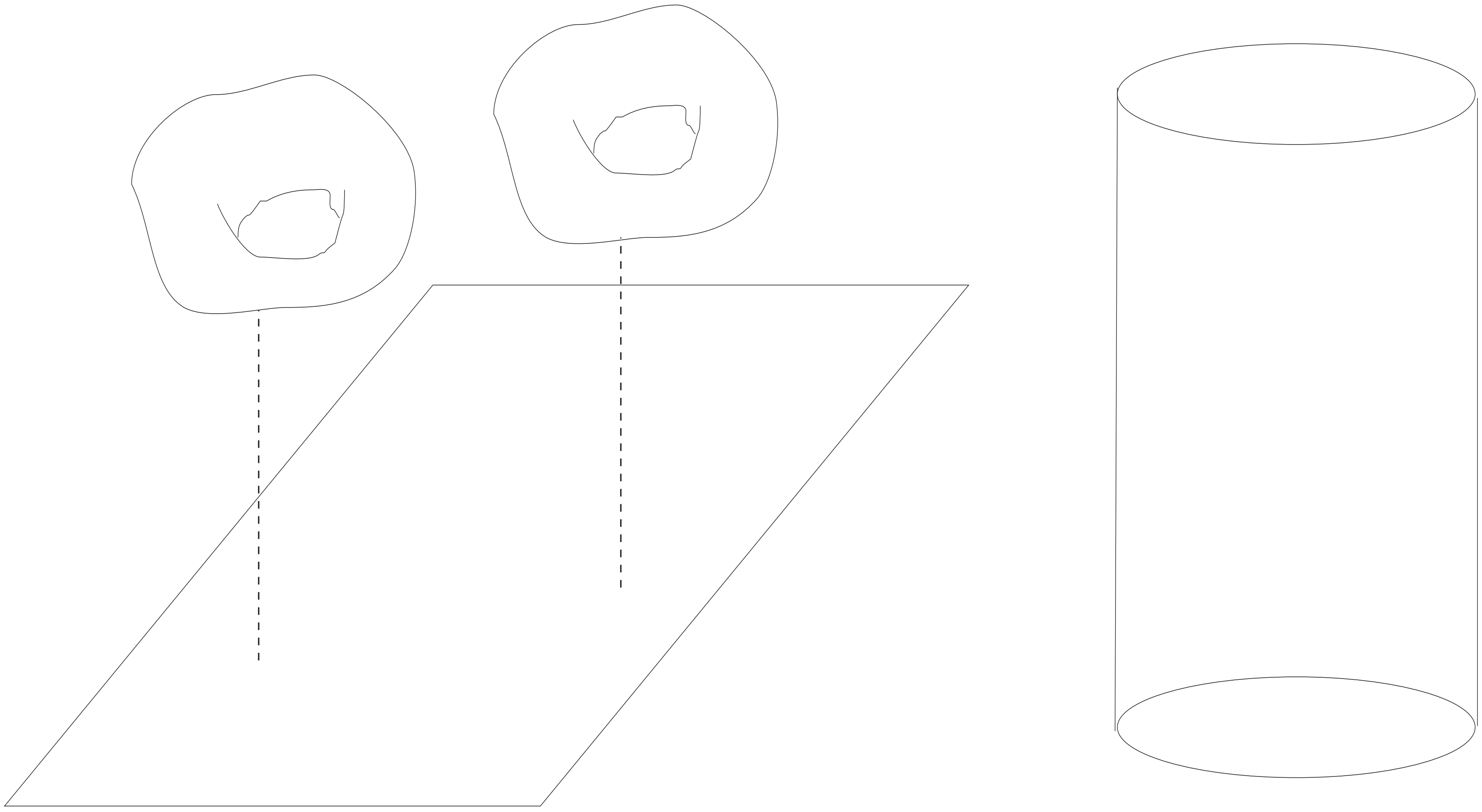
$$V(r) \sim \frac{1}{r^{d-2}}$$

Extra Dimensions

Seems to be 2D surface at large distances but has pits at shorter once.

Kaluza-Klein Compactification

Spacetime $= \mathbb{R}^{1,3} \times \boldsymbol{K}^n$

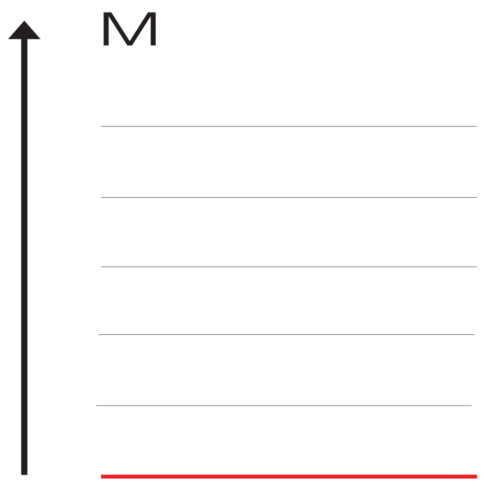


Kaluza-Klein Compactification

Massless scalar field $S=\int\limits_{-\infty}^{+\infty} dz \int d^4x\, \partial_A\phi\, \partial^A\phi$

$\phi(x,z+2\pi R)=\phi(x,z)$, expand $\phi(x,z)=\sum_{-\infty}^{+\infty}\phi_n(x)e^{in\frac{z}{R}}$
Action

$$S=\sum_{-\infty}^{+\infty}\int d^4x\left(\partial_A\phi_n\partial^A\phi_{-n}-\frac{n^2}{R^2}\phi_n\phi_{-n}\right)$$



KK - tower of states

Kaluza-Klein

Spectrum $E_n^2 = m_n^2 = n^2/R^2$
Field equation (Klein-Gordon)

$$\left[\partial_A \partial^A - m_n^2\right] \phi_n(x,z) = 0 \Longrightarrow \left[-\partial_z^2 + \mathrm{p}^2 + m_n^2\right] \phi_n(z) = E_n^2 \phi_n$$

Green function $(p=0)$

$$\hat{V}(r) = \sum \frac{\phi_n(0)\phi_n^*(0)}{E_n^2}$$

One has $\phi_n(x) = e^{-m_n|x|}/|x|$

$$V(r) = \frac{1}{r} + \frac{e^{-r/R}}{r} + \ldots$$

Alternative to Compactification

BUT! The radius $\boldsymbol{R}$ should be too small to recover Newton law in 4D. Almost impossible to verify experimentally.

Compact extra dim $\Longrightarrow$ Noncompact (infinite)

$\mathbb{R}^{1,3} \times K^n \Longrightarrow$ Warped space

$$M = N(x) \times K(y)$$

$ds^2 = ds_N^2(x) + ds_K^2(y)$

This is not the case for warped product

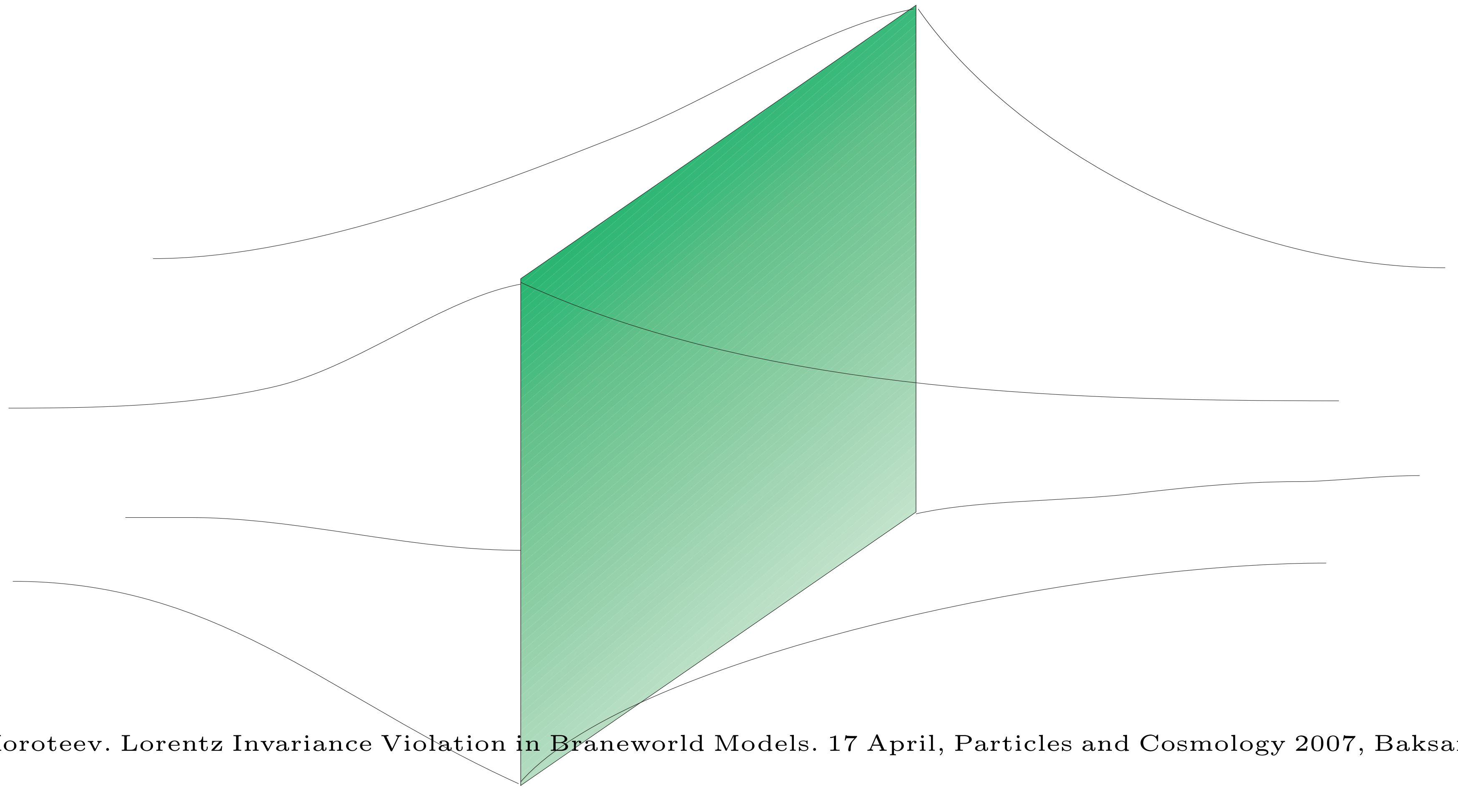
$ds^2 = ds_1^2(x,y) + ds_2^2(x,y)$

e.g. AdS_5

$$ds^2 = \frac{R^2}{y^2}\left(dt^2 - d\mathrm{x}^2 - dy^2\right)$$

Alternative to Compactification

Introduce topological defect – 3 brane [Randall, Sundrum '99] $G_{AB} = T_{AB}^{(bulk)} + T_{AB}^{(brane)}$ for $T_{AB}^{(brane)} = \mathrm{diag}(\varepsilon_b\, p_b\, p_b\, p_b\, 0)\delta(y - y^{(brane)})/\sqrt{g}$



Braneworld

$$S = S_{gravity} + S_{brane} = \int d^5x\sqrt{g}\left(R - 2M^3\Lambda + \delta(z - z^{(brane)})\mathscr{L}\right)$$

Possible solution

- Solve bulk EE without brane

$$G_{AB} = T_{AB}^{(bulk)}$$

- Impose junction conditions at the location of the brane. Peterson–Codazzi eqns $\Rightarrow$ junction conditions [Israel '66]

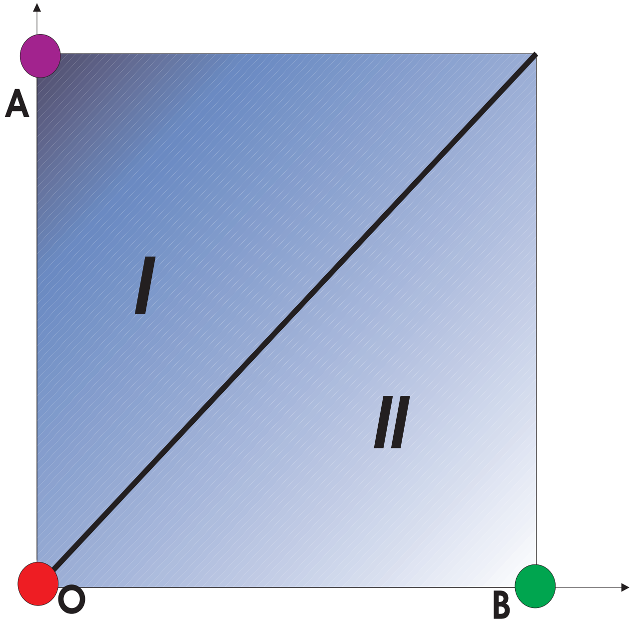
$$[K_B^A]|^{(brane)} = S_B^A|^{(brane)} - \frac{1}{3}S^{(brane)}\delta_B^A$$

1 Extra Dimension

$SO(3) \times \mathbb{Z}_2$ symmetric background can be written as

$$ds^2_{(\xi,\zeta)} = e^{2\xi k|z|}dt^2 - e^{2\zeta k|z|}d\mathrm{x}^2 - dz^2$$

Violation of Lorentz invariance in the bulk. Iff $z = 0$ do we have $SO(1,3) \times \mathbb{Z}_2$-symmetry.



1 Extra Dimension. Matter

Ideal relativistic fluid in the bulk

$$T^A_B = u^A u_B \rho - p\delta^A_B + \Lambda, \quad u_A u^A = 1$$

Equation of state $p = w\rho$, $p_5 = \omega\rho$. Null Energy condition $|w| < 1, |\omega| < 1$

$$\rho = -\Lambda + 6k^2\zeta^2$$

$$w = -1 + \frac{3\zeta^2 - 2\zeta\xi - \xi^2}{\rho}$$

$$\omega = -1 + \frac{3\zeta(\zeta - \xi)}{\rho}$$

Null Energy Condition (NEC)

$$T_{AB}\xi^A\xi^B \geq 0, \quad g_{AB}\xi^A\xi^B = 0$$

Can be treated as minimization the bilinear form T over the hypersurface $g_{AB}\xi^A\xi^B = 0$

For simple equation of state $p = w\rho$ NEC reads $w \geq -1$ For vacuum $w = -1$. Each reasonable matter is supposed to satisfy the NEC.

It appears that $w < -1$ is not too bad sometimes...(Phantoms...)

No-Go Theorem

Irrespective of matter in question:

Let the spatial curvature of the brane be equal to zero. Then one cannot screen bulk naked singularity from the brane by means of horizon if NEC on the brane and in the bulk are satisfied.

The "exceptional"case is the AdS space

$$ds^2_{(D+1)} = e^{-k|z|}ds^2_{(D)} - dz^2$$

warped sliced space – the solution of braneworld setup with Λ-term in the bulk and λ-term on the brane.
NEC is satisfied: $\omega = w = -1$
and the relation from junction conditions
$F(k, \Lambda, \lambda) = 0$

The way out – closed geometry

Flat spartial $D-1+1$ brane $\Rightarrow$ Sphere$\times$ time
$S^{D-1}\times\mathbb{R}^1$

$$ds^2 = dt^2 - \frac{e^{-2k|z|}}{\left(1+\frac{\kappa^2}{4}x_i x^i\right)^2} d\mathrm{x}^2 - dz^2,$$

The solution for matter

$$w = \frac{1}{3}\frac{e^{2kz}-3k^2/\kappa^2}{e^{2kz}-3k^2/\kappa^2}, \qquad \omega = \frac{e^{2kz}-k^2/\kappa^2}{e^{2kz}-2k^2/\kappa^2}$$

Thus if k/κ is small enough NEC is satisfied!

Spectrum of gravitons (TT)

KG equation

$$\left[\partial_z^2 + E^2 e^{2\xi k|z|} - p^2 e^{2\zeta k|z|} - (\xi + 3\zeta)\partial_z\right]\phi(z) = 0$$

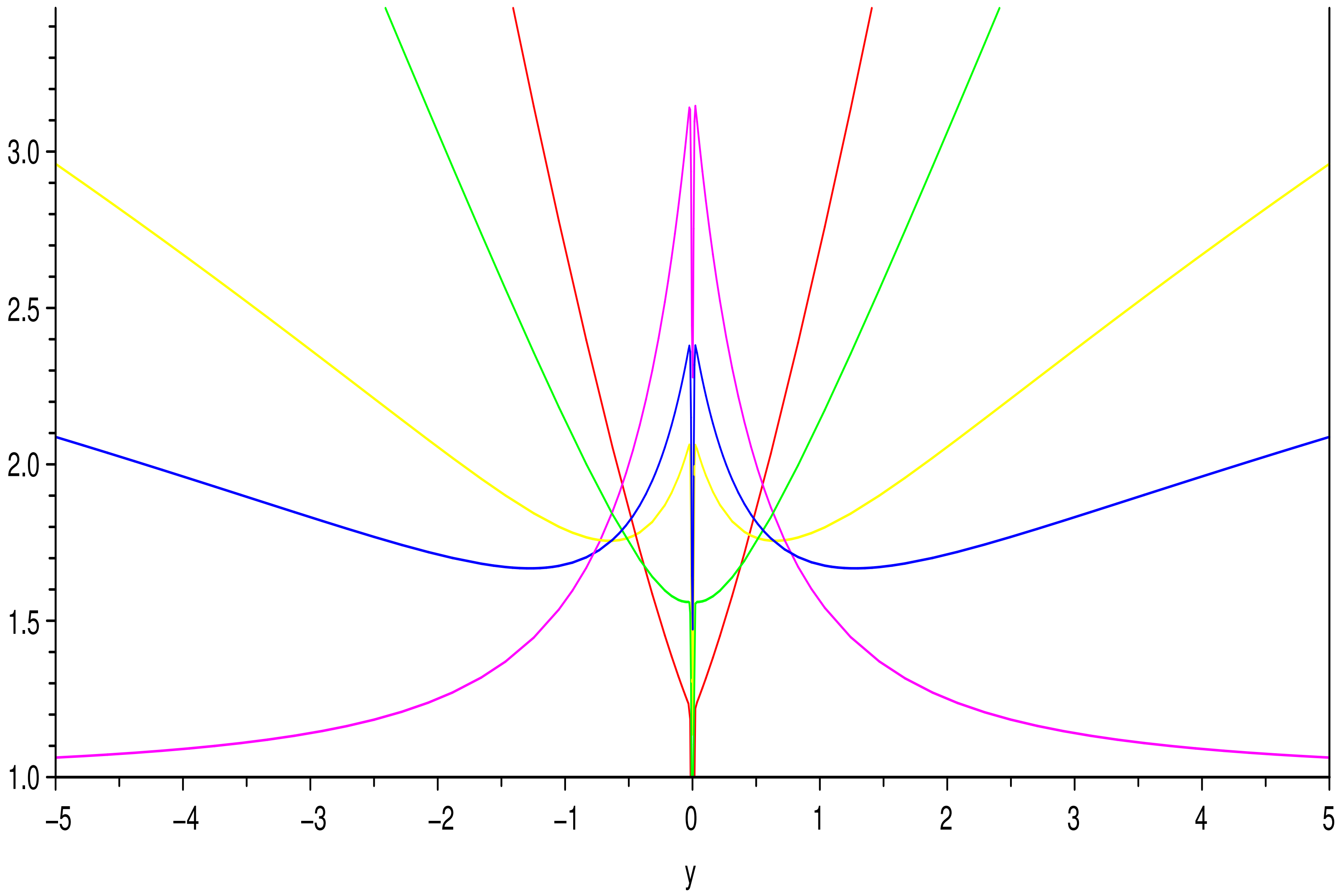
Schrödinger equation

$$\left[\partial_z^2 + (\mathscr{E} - V_p)\right]\phi(y) = 0$$

with $\mathscr{E} = E^2$ in the potential

$$V_p(y) = p^2\left(1 + \xi k|y|\right)^{2(1-\zeta/\xi)} + \frac{9\zeta^2 k^2}{4\left(1 + \xi k|y|\right)^2} - 3\zeta k \delta(y)$$

The family of effective potentials



Effective Potential

Traceless transverse component of metric excitations coincides with scalar one

Relation between 5d Plank constant and 4d one

$$M_{(4)}^2 = M_{(5)}^3 \int\limits_{-\infty}^{+\infty} dz\, e^{-\frac{\xi+3\zeta}{2}k|z|} \sim \frac{M_{(5)}^3}{k}$$

Gravitational potential

$$V(r) = \frac{1}{M_{(5)}^3} G(r)$$

Spectrum. Randall–Sundrum

$\xi = \zeta = 1$
Decaying potential $\Longrightarrow$ quasilocalized states,
continuous spectrum
Gravitational potential

$$V(r) \sim \frac{1}{r}\Big(1 + \frac{1}{k^2 r^2}\Big)$$

The same situation in II triangle

Type I Model

$\xi = 0$, $\zeta = 1$
Box type potential, discrete spectrum, zero mode with dipertion relation $E^2 = 3p^2 + 3kp$

$$E_n^2 \approx \frac{9}{4}k^2 + \frac{Cn^2}{\log p/k}$$

Gravitational potential

$$V(r) \sim \frac{1}{r}\left(1 + \frac{1}{\pi k r}\right)$$

All excited modes are localized on the brane

Outlook

- Dynamic solution $\Rightarrow$ Cosmology
- Holography. How will the principle will change if the conformal theory is broken? "AdS"/"CFT correpondence"