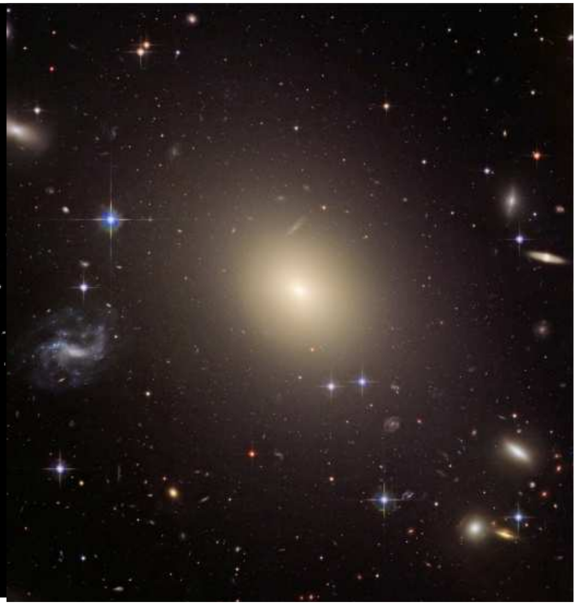

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A solution of the cusp problem in virialized DM halos

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Topics

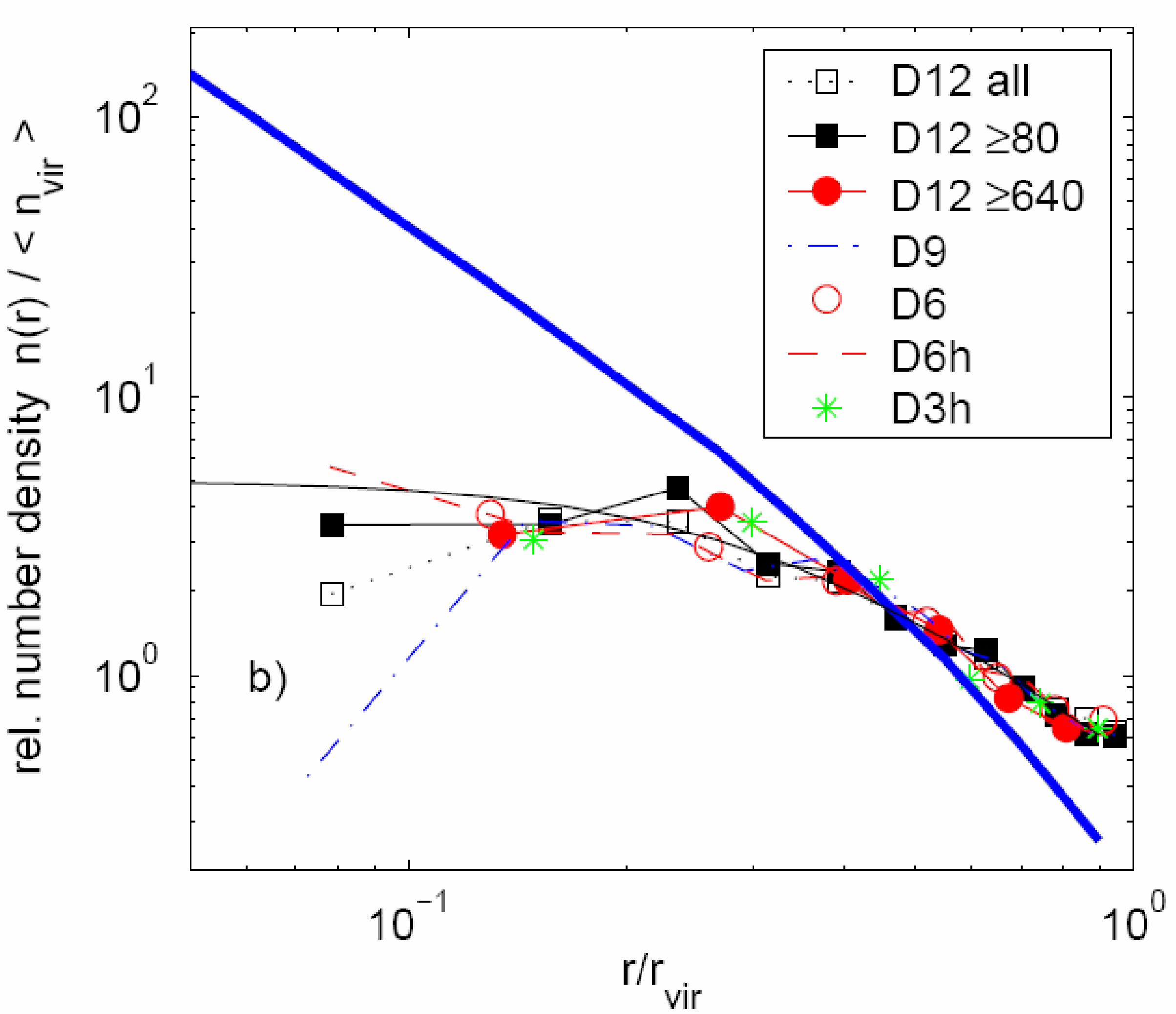
Cusp problem

Situation before our research
(observations and simulations)

Solution of cusp problem

Conclusions

Cusp problem



Diemand et al. 2004

Density profiles

$$\rho(r) \propto r^{-\alpha} \quad \text{at} \quad r \rightarrow 0$$

observations

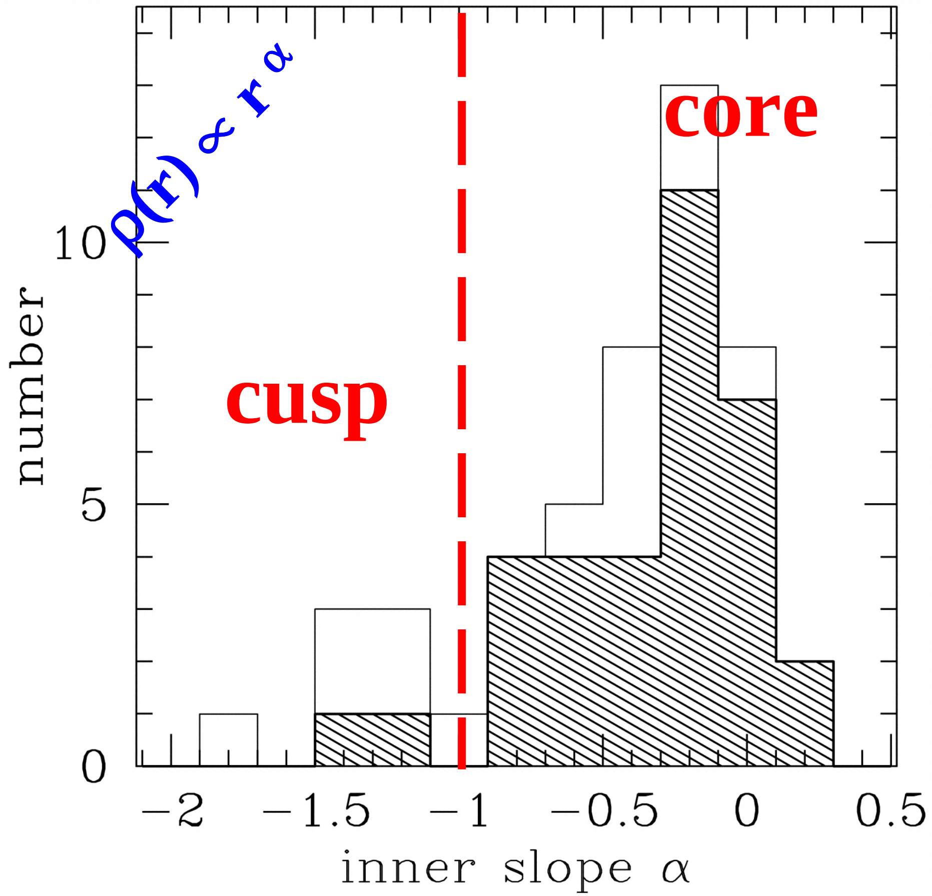
$\alpha < 1$ - **core** (***finite*** pressure in the centre)

$\alpha \geq 1$ - **cusp** (***infinite*** pressure in the centre)

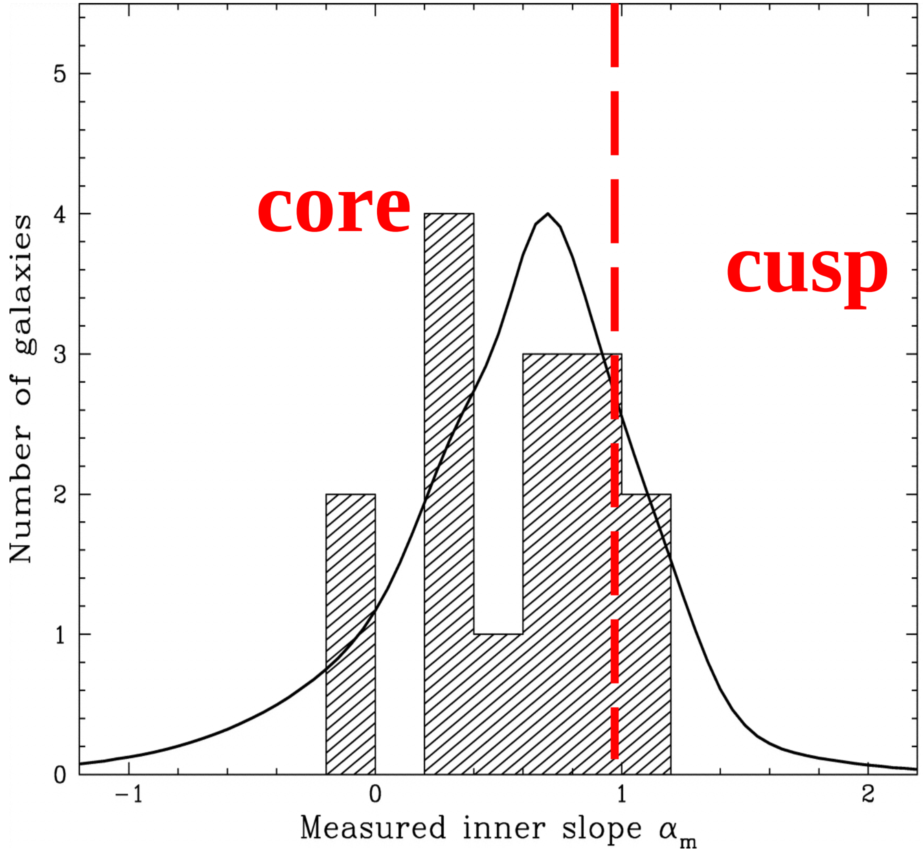
N-body simulations

Best observations

48 low brightness galaxies
(de Blok et al., 2001)



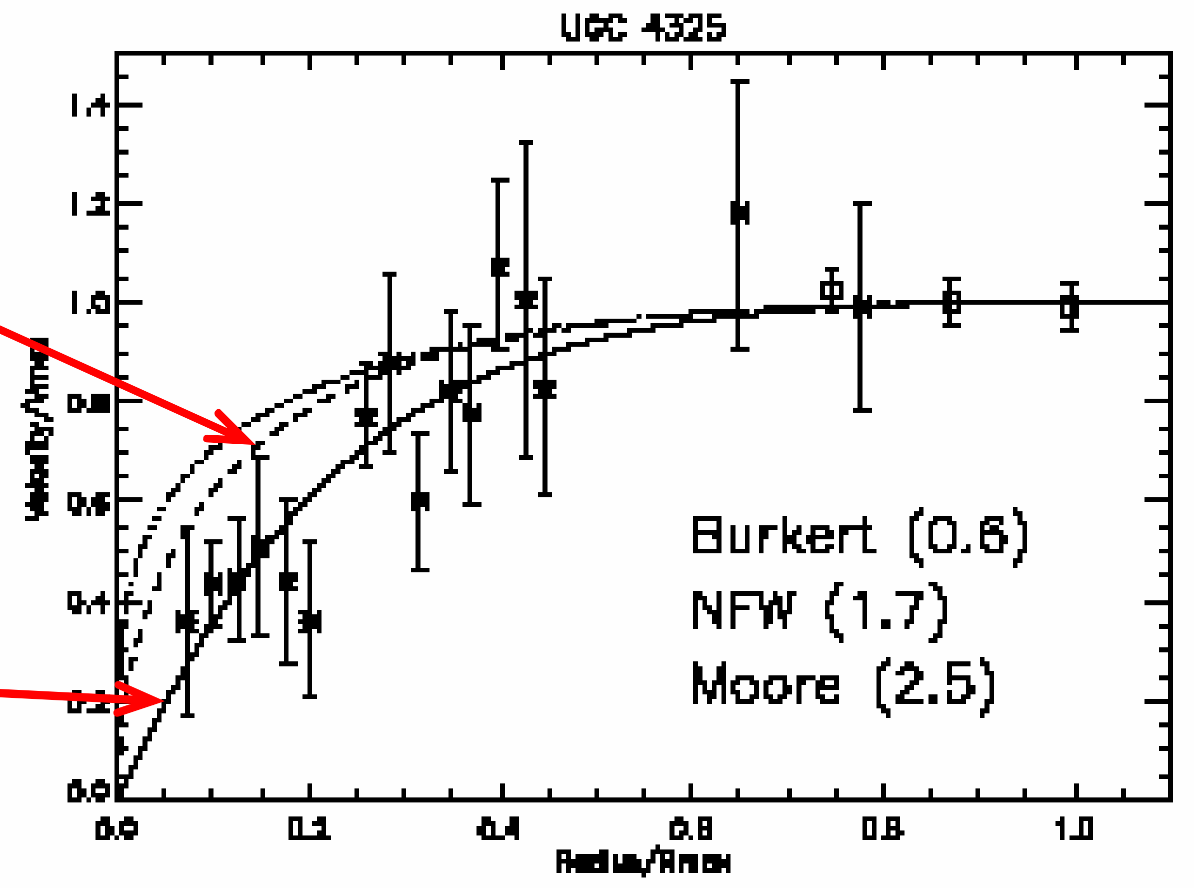
15 low brightness galaxies
(Swaters et al., 2003)



Rotation curve

Dashed line –
NFW profile:
$P(x) \sim x^{-1}(1+x)^{-2}$

Solid line –
Burkert profile:
$P(x) \sim (1+x)^{-1}(1+x^2)^{-1}$



(Marchesini 2002)

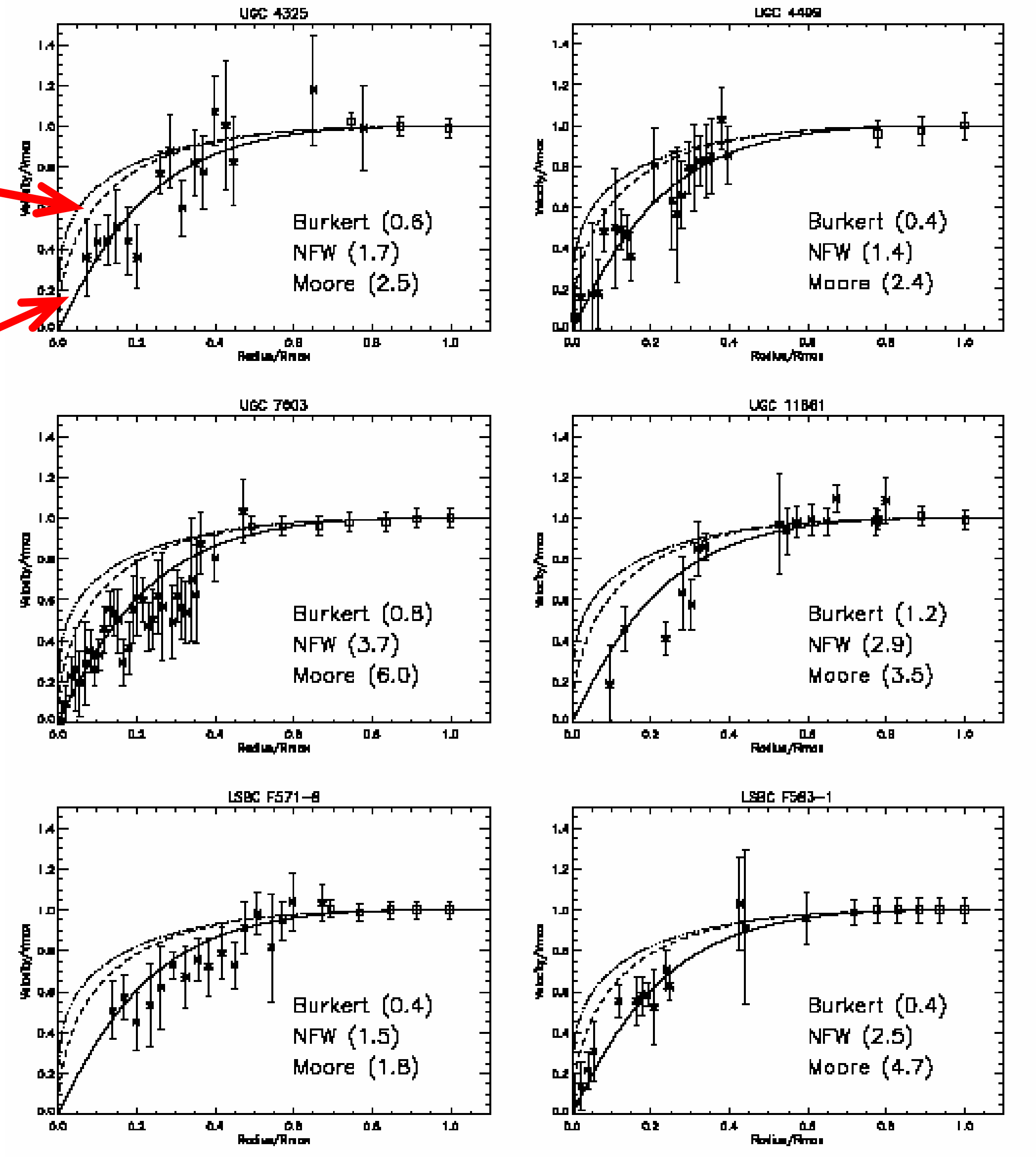
$x = r/r_s$

Rotation curves

Dashed line – NFW profile: $\rho \sim x^{-1}(1+x)^{-2}$

Solid line – Burkert profile: $\rho \sim (1+x)^{-1}(1+x^2)^{-1}$

$x = r/r_s$



(Marchesini 2002)

Does nature create cusps?

Does nature create cusps?

Our answer: NO

The problem of cusp can be solved in the framework of standard ΛCDM

Core instead of cusp

Idea: to take into account ***the small scale*** part of initial background perturbations that transforms into random velocities of DM particles in the process of relaxation

Method: **total** *entropy* = **initial** (given) + **generated** (gained during relaxation)

Entropy profiles related to density profiles in DM halos

Entropy function

*for DM particles with isotropic velocity distribution in **relaxed** halos*

$$p = \rho\langle v^2\rangle = nT = Fn^{5/3}$$

$\langle v^2\rangle$: **one-dimensional peculiar velocity**

F: **entropy function**

Power-law density profiles: $\rho(r) \propto r^{-\alpha}, \quad \alpha \in (0, 2)$

$$M = M(r) = \int_0^{\infty} \rho(r) r^2 dr \propto r^{3-\alpha}$$

+ Hydrostatic equilibrium:

$$\frac{1}{\rho}\frac{dp}{dr} = -\frac{GM(r)}{r^2}$$

$$p = C_1 + C_2 r^{2(1-\alpha)}$$

$$\rho(r) \propto r^{-\alpha}, \quad p = C_1 + C_2\, r^{2(1-\alpha)}$$

$\alpha = 1$ is a critical value

$0 < \alpha < 1$ - **core** (***finite*** pressure in the centre)

$1 \leq \alpha < 2$ - **cusp** (***infinite*** pressure in the centre)

$r \to M$ **conserving both for initial and relaxed matter fields**

Entropy mass function

$$F(M) \propto C_1 M^{\beta_1} + C_2 M^{\beta_2} \propto M^{\beta}$$

$$\beta_{1,2} = \frac{1 + 2\alpha/3}{3 - \alpha} \pm \frac{|\alpha - 1|}{\alpha - 3}, \quad \beta \in (\beta_1, \beta_2)$$

$$\alpha_{cr} = \boxed{1} \Rightarrow \quad \beta_{cr} = \beta_1 = \beta_2 = \boxed{\frac{5}{6}}$$

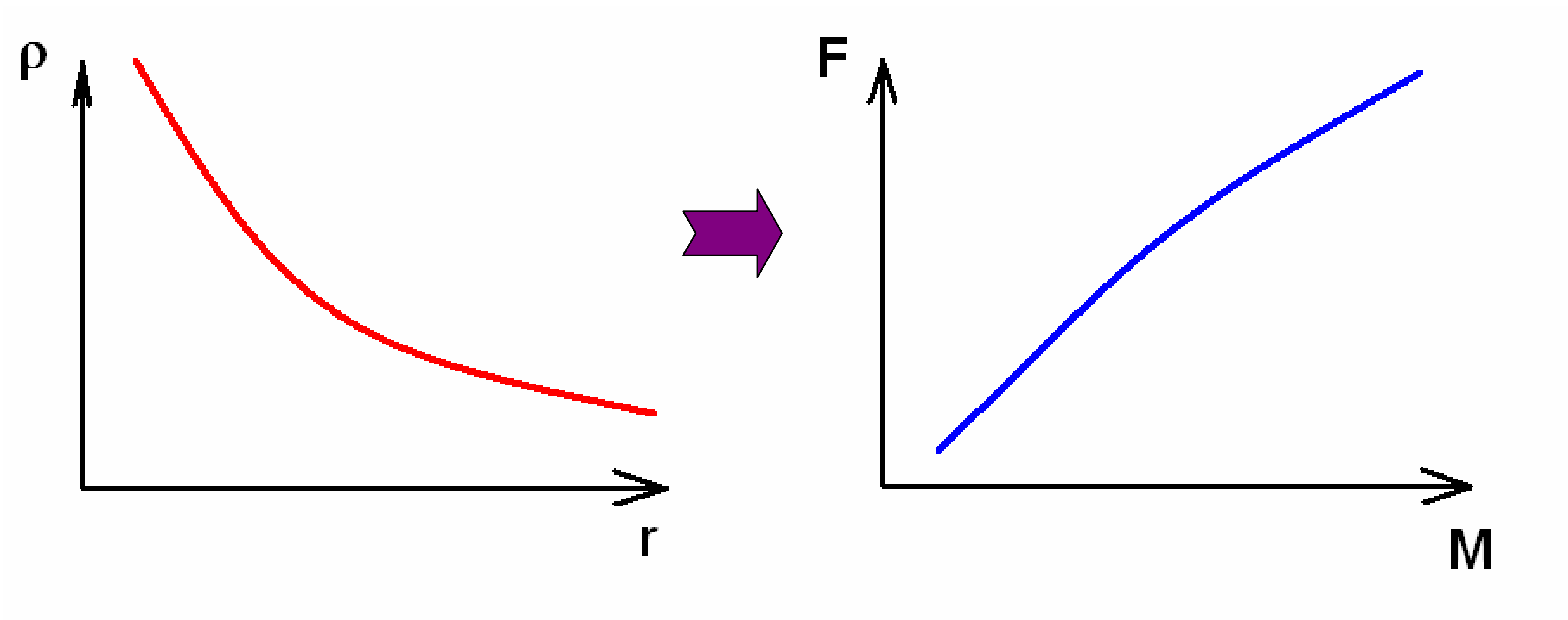
core : $\beta_1 \in \left(0, \frac{5}{6}\right)$, $\beta_2 \in \left(\frac{2}{3}, \frac{5}{6}\right)$

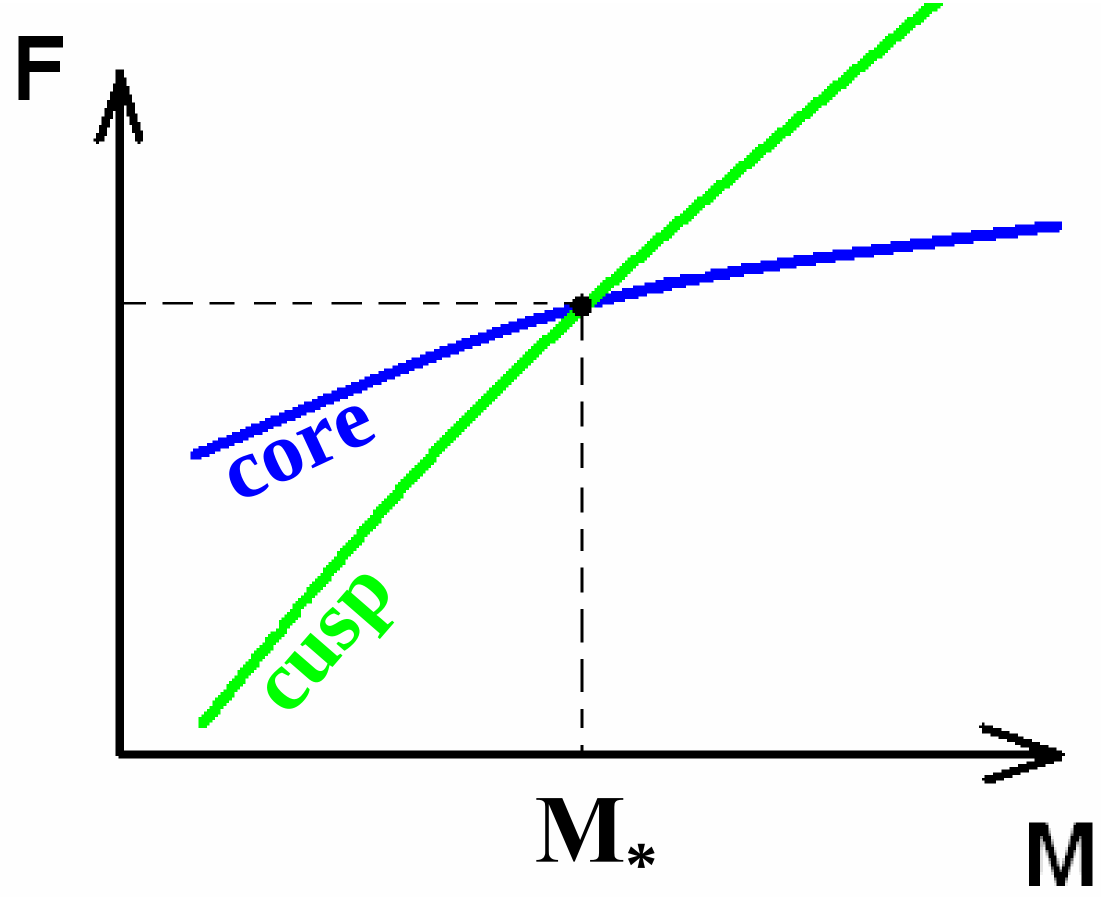
$\alpha = 0$

$\alpha = 2$

cusp: $\beta_1 \in \left(\frac{5}{6}, \frac{4}{3}\right)$, $\beta_2 \in \left(\frac{5}{6}, \frac{10}{3}\right)$

Density profile ⇨ Entropy mass function




F
core
cusp
M*
M

Calculations have been done in standard cosmological model:

- $H_0 = 70$ km/s/Mpc , $h = 0.7$
- $\Omega_\Lambda = 0.7$
- $\Omega_m = 0.3$
- ΛCDM power spectrum:

$$P(k) = A\,\kappa T^2(\kappa)\exp(-R_f^2\,\kappa^2), \quad \kappa = k/k_0$$

$$k_0 = 0.2\ h/\text{Mpc}, \quad A = 240\,\sigma_8^2/k_0^3$$

$$\frac{H(z)}{H_0} = \sqrt{1 + \Omega_m (z^3 + 3z^2 + 3z)} \Rightarrow 0.5(1+z)^{3/2}, \; z > 1$$

$$\rho_m = \frac{3H_0^{\;2}}{8\pi G} \Omega_m (1+z)^3 \cong 3 \cdot 10^{-30} (1+z)^3 \frac{g}{cm^3}$$

$$n = \frac{\rho_m}{m_{DM}} \cong 3 \cdot 10^{-6} \mu^{-1} (1+z)^3 \, cm^{-3}, \quad \mu \equiv \frac{m_{DM}}{m_p}$$

R_f - particle free path in the early Universe

$m_{DM} > 1 M эB, \quad R_f < 4 \cdot 10^{-6}$ - star size halos

Initial entropy

Linear field of density perturbations

$$a = 1$$

- Displacement $\vec{S} = \vec{r} - \vec{x}$
- Velocity $\vec{V} = \dot{\vec{r}}$
- Density perturbation $\delta = div\,\vec{S}$
- Coordinates:

Euler $(t, \vec{r})$

Lagrange $(\tau, \vec{x})$

$$(1+2\Phi)dt^2 - a^2(1-2\Phi)d\vec{r}^{\,2} =$$

$$= d\tau^2 - a^2\left((1-2q)\delta_{\alpha\beta} - 2B_{,\alpha\beta}\right)dx^\alpha dx^\beta$$

DM halo formation

(Zel'dovich approximation)

$$\vec{r}(z,\vec{x}) = (1+z)^{-1}[\vec{x} - g(z)\vec{S}(\vec{x})] , \quad \sigma = \sqrt{\langle \vec{S}^2 \rangle} = 11 h^{-1} Mpc$$

$$\vec{V}(z,\vec{x}) \cong H_0 (1+z)^{1/2} [\vec{x}/2 - g(z)\vec{S}(\vec{x})]$$

$$\vec{S}(\vec{x}) = \vec{S}_R(\vec{x}) + \vec{S}_*(\vec{x}), \; \delta(\vec{x}) = \delta_R(\vec{x}) + \delta_*(\vec{x})$$

local background – protohalo with linear scale R [collapsing into virialized halo by z_0 with compression factor ~ $5(1+z_0)$]

conditional perturbations [transform (adiabatically at least) into microscopic particles' motion in halo]

$$|\vec{x} - \vec{x}_0| < R: \quad \delta_R \cong 1.69(1+z_0), \quad \vec{S}_R(\vec{x}) \cong \vec{S}_R(\vec{x}_0)$$

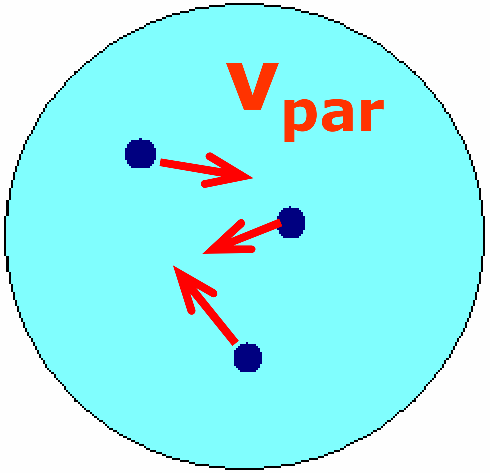
$$\langle \vec{S}_* \rangle = \langle \delta_* \rangle = 0 \;, \quad \langle \vec{S}_*^2 \rangle \equiv \sigma_*^2(R) = \int P(k)[1 - W(kR)]^2 dk$$

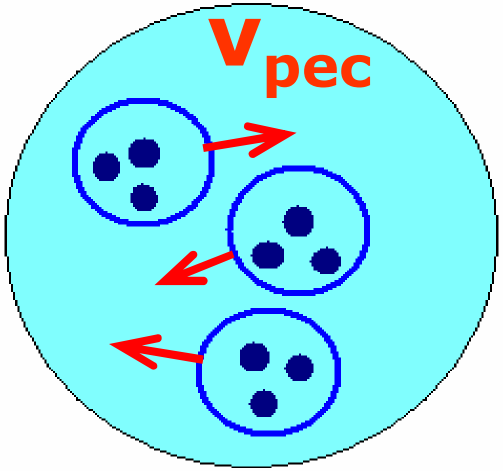
Entropy ⇒ assemble

ideal gas | **collapsing cloud**

R > particle separation ⇒ perturbation size

t > R/v_{par} ⇒ collapse time, t_0





at $t < t_0$ we average over assemble of clouds collapsing by the given t_0

Conditional variance of peculiar velocity inside halo of mass M

$$\vec{v}_*(z,\vec{x}) \cong H_0(1+z)^{-1/2}\vec{S}_*(\vec{x})$$

$$\sigma_{v*}^2 \equiv \sigma_{v*}^2(z,M) \cong H_0^2(1+z)^{-1}\sigma_*^2(R)$$

$$\sigma_* / \sigma \approx M_{13}^{1/3}\, ln[\, 1 + M_{13}^{-1/3}\,]$$

$$R \cong \sigma M_{15}^{1/3}\ ,\quad M_n = M / 10^n\ M_{\odot}$$

$$\langle F(M)\rangle = \frac{m_{DM}\sigma_v^2(z_0,M)}{3n^{2/3}(z_0)} \approx F_0 z_5^{-3} M_{10}^{2/3} \ln^2[1+M_{13}^{-1/3}]$$

$$M_{13} = M/10^{13}M_{\odot}$$

$$F_0 = \mu^{5/3} keV\,cm^2,\; z_5 \equiv (1+z_0)/5,\; \mu \equiv m_{DM}/m_p$$

Probability distribution function

$$dW(f) = e^{-f/2}\frac{df}{\sqrt{2\pi f}}, \quad f = \frac{F(M)}{\langle F(M)\rangle}$$

$$\langle f^2\rangle = 3\langle f\rangle^2 = 3$$

large variations of F from mean value

$$\beta \equiv \frac{d\ ln\ F(M\ )}{d\ ln\ M} = 2\left(\frac{1}{3} + \frac{1}{ln\ M_{13}}\right)$$

$$n \equiv log\ (M\ /M_{\odot})$$

n	10	7	4	0
β	0.3	0.5	0.57	0.67

$$< \frac{5}{6} \cong 0.83$$

Critical value for β

Generated entropy

Violent relaxation entropy (special analytical models)

Isotermal shere

(Fillmore & Goldreich 1984)

$$\rho \sim r^{-2}, \quad M \sim r, \quad F_g \sim M^{4/3}$$

Collapse of ellipsoide

(Gurevich, Zybin 1988, 1995)

$$\alpha \sim 1.7 - 1.9$$

Generated entropy in the central region is negligible in comparison with background one!

Total entropy

- Background entropy from small scale perturbations

$$F_b \sim M^{1/3-2/3}$$

- Generated entropy from violent and hierarchical relaxation of compressed matter

$$F_g \sim M^{5/6-4/3}$$

Analytically modeled halos

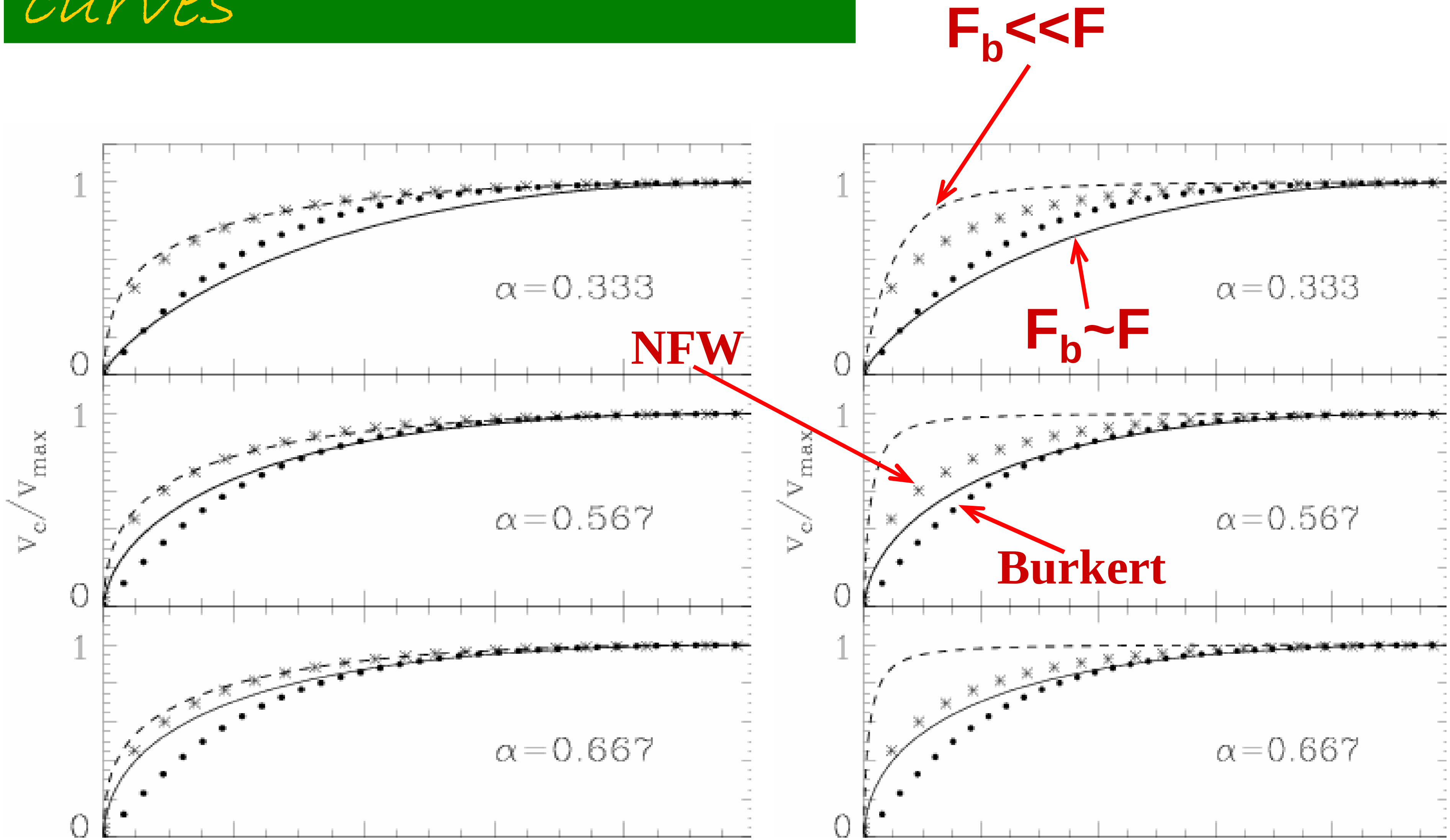
$$F_b(M) \sim M^{\beta_b}, \qquad \beta_b < 5/6$$

$$F_g(M) \sim M^{\beta_g}, \qquad \beta_g \geq 5/6$$

$$F(M) = \sqrt{C_b M^{2\beta_b} + C_g M^{2\beta_g}}$$

$$\kappa = \frac{F_b}{F} \in (0,1)$$

Generated rotation curves



β_g =5/6 (NFW) r/r_{max} β_g =4/3 (isothermal sphere)

Conclusions

* The background entropy can prevent the cusp formation for halos with

$$10^6\, M_\odot < M < 10^{12}\, M_\odot$$

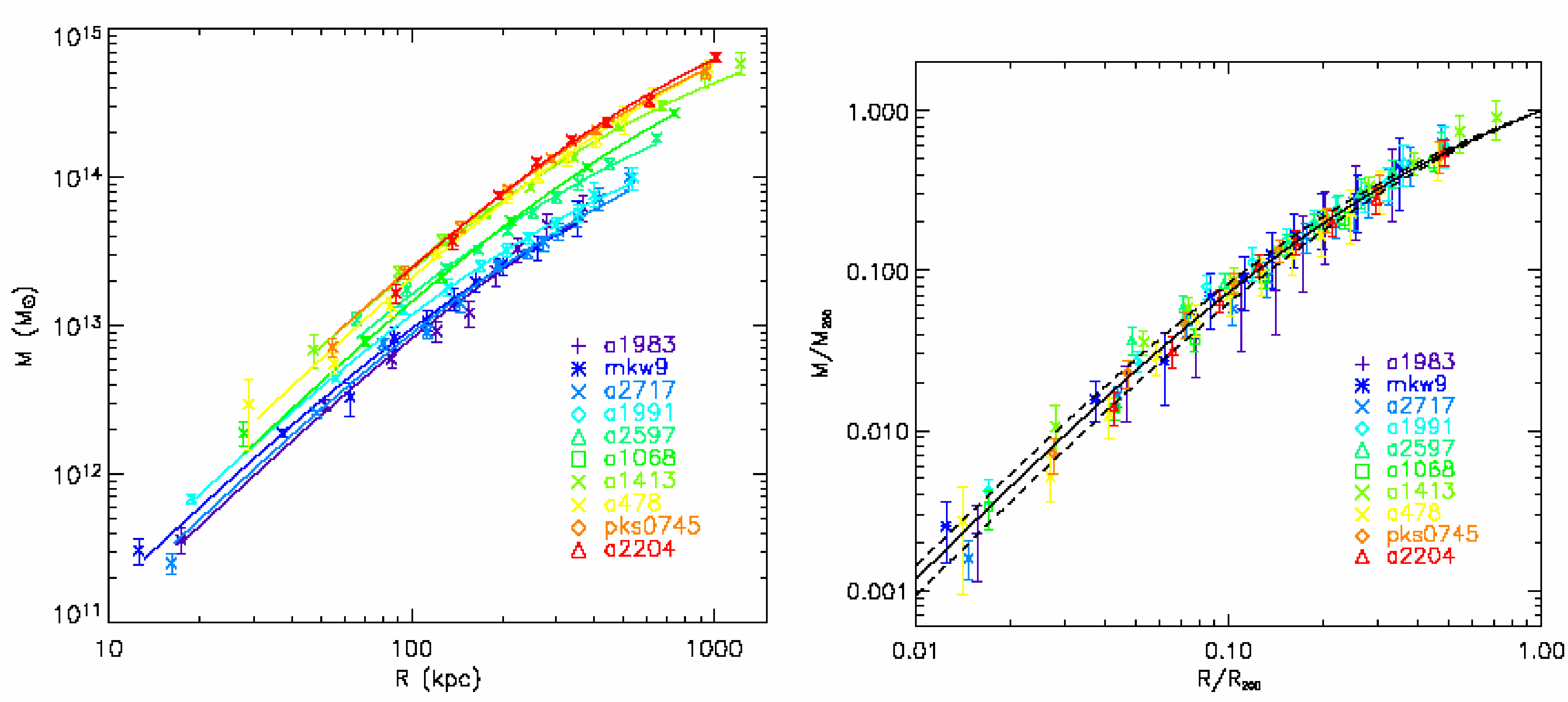
* For smaller and larger galaxies and for clusters of galaxies the impact of background entropy is attenuated

* The impact of the background entropy allows to reproduce the observed rotation curves

* N-body simulations: underestimation of initial perturbations at small scale?

Additional slides

Galaxy clusters



(Pointecouteau et al., 2005)

Equilibrium DM halo

- **Adiabatic and irreversible processes**
- **Entropy function:** $F = T/n^{2/3} = p/n^{5/3}$
- **Hydrostatic equilibrium:**

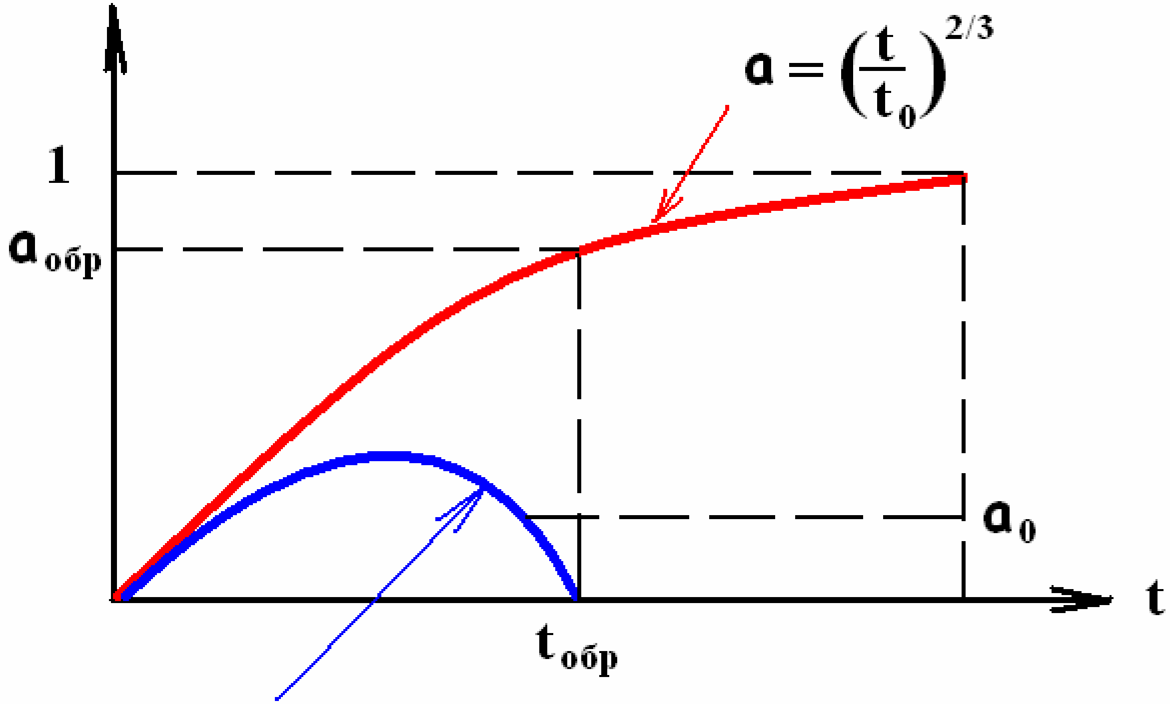
$$\frac{1}{\rho}\frac{dp}{dr} = -\frac{GM(r)}{r^2}$$

- **Initial (background) entropy:**

$$F \sim M^{1/3-2/3}$$

- **Hierarchical (mergers) and violent relaxation entropy of compressed matter:**

$$F \sim M^{5/6-4/3}$$



$$\begin{cases} a = a_0\left(1 - \cos\dfrac{\eta}{\eta_0}\right) \\ t = a_0\left(\eta - \eta_0 \sin\dfrac{\eta}{\eta_0}\right) \end{cases}$$

Cusp realization needs low entropy particles

- **How to transport the low energy particles into the cusp (the compression factor is limited)?**
- **Where to take low entropy particles in real density fields?**

CDM is cold, but there are perturbations at small scale

CDM preheated by primordial perturbations in collapsing protohalos

N-body: Is there a refrigerator at small scales?

Hierarchical clustering/merging (N-body simulations)

- **Density and entropy slopes in halos:**

$$\rho \sim r^{-\alpha}, \quad F \sim M^{\beta}$$

- **Universal NFW profile:**

$$\rho \sim x^{-1}(1+x)^{-2}$$

$$x = r/r_s, \; \alpha = 1, \; \beta = 5/6$$

- **Empirical Burkert profile:**

$$\rho \sim (1+x)^{-1}(1+x^2)^{-1}$$

$$\alpha = 0, \; \beta = 0$$

$$\left\langle F(M)\right\rangle = \frac{m_{DM}\sigma_v^2(z_0,q)}{3n^{2/3}(z_0)} \approx F_0 z_5^{-3} \begin{cases} M_9^{0.33}, & M_9 > 1 \\ M_9^{0.56}, & M_9 < 1 \\ 10^{-5} M_0^{0.66}, & M_0 \le 1 \end{cases}$$

$$F_0 = \mu^{5/3} keV\, cm^2 \;,\; z_5 \equiv \frac{1+z_0}{5} \,,\; M_n = \frac{M(q)}{10^n M_S}$$