Towards event-by-event composition studies

GZK-40

The Third International Workshop on
Ultra-high-energy cosmic rays and their sources

INR, Moscow, May 16-18, 2006

Dmitry S. Gorbunov
D.S.G., G.I. Rubtsov, S.V. Troitsky

gorby@ms2.inr.ac.ru

Institute for Nuclear Research, Moscow

Motivation

What can we learn from GZK-studies,
if the cutoff would be observed?

Astrophysics	Particle physics
sources:	interactions at $E_{cm} \gtrsim 300$ TeV:
accelaration mechanism,	cross section,
local backgrounds	inelasticity,
space:	multiplicity,
γ-background,	P_T-distribution,
magnetic fileds	new physics phenomena?

Motivation

What can we learn from GZK-studies,
if the cutoff would be observed?

Astrophysics	Particle physics
sources:	interactions at $E_{cm} \gtrsim 300$ TeV:
accelaration mechanism,	cross section,
local backgrounds	inelasticity,
space:	multiplicity,
γ-background,	P_T-distribution,
magnetic fileds	new physics phenomena?

chemical composition

Plan

- The main idea: event-by-event analysis
- Chemical composition from observed events with “estimated” primary type
- Several Examples: highest energy AGASA and Yakutsk events
- How one can estimate the primary of a given observed event
- What else?

Event-by-event analysis

It is obviously important:

- just several parameters of EAS are measured
- significant fluctuations
- degenerecy: similar parameters for different primaries
- azimuth angle dependence
- global anisotropy?
- evolution of chemical compoistion with energy
- poor statistics

Interesting questions: a given event

For a given observed (several parameters of EAS are measured) event:

- conservative study (negative knowledge): what is the probability, that it could not be initiated by the primary A?
- (positive knowledge): what is the probability, that it could be initiated by a primary of a given type A_i (say, $A_i = p, \gamma, \ldots,$ Fe)?
- ...
- what is the most probable initial energy, if the primary was A_i?
- (z, θ) are fixed ($contra$ examples?)

Interesting questions: combined set

Chemical composition of UHECR within a given energy interval $E_{min} < E < E_{max}$:

- (negative knowledge): the upper limit on the primary A
- (positive knowledge): what is the most probable chemical composition (the set of possible primaries is fixed)?
- ...
- selection cuts
- global (an)isotropy

Procedure to estimate the chemical composition

1. one selects the set of observed events (EAS parameters have been measured) — experimental cuts ($z < 45^\circ$), quality cuts?
2. for each event j one compares the parameters of simulated EAS of various primaries to parameters of observed EAS and estimates the probabilities $p_i(j)$ that it could (not) be initiated by a primary A_i of energy within $E_{min} < E < E_{max}$ ($\equiv \mathcal{E}$)
3. $p_i(j)$ enables one to estimate the probability that among N events n_i were (not) initiated by primary A_i — combinatorics
4. one estimates the most probable chemical composition ϵ_i or set the upper bound on a primary A, which are consistent with selected set of observed events — likelihood
5. one takes into account possible corrections because of cuts on initial set, global anisotopy, etc.

The upper limit on a fraction of primary A

1. Input

set of N observed events with energies $\mathcal{E}$,

$p_A^{(+)j}$, $p_{\overline{A}}^{(+)j}$, generally $p_A^{(+)j} + p_{\overline{A}}^{(-)j} \neq 1$

Steps

2. Probability $\mathcal{P}(n_1, n_2)$: among N observed events, n_1 initiated by A with $\mathcal{E}$ and n_2 initiated by $\overline{A}$ with $\mathcal{E}$:

The probability to have i_1-th, . . . , i_{n_1}-th events ($i_1 < \cdots < i_{n_1}$) induced by A with $\mathcal{E}$ and k_1-th, . . . , k_{n_2}-th events ($k_1 < \cdots < k_{n_2}$, $i_j \neq k_l$) induced by $\overline{A}$ with $\mathcal{E}$:

$$\mathcal{P}\left(\{i_j\},\{k_l\}\right) = \prod_{i_j} p_A^{(+)i_j} \prod_{k_l} p_{\overline{A}}^{(+)k_l} \prod_{m_n \neq i_j, k_l} \left(1 - p_A^{(+)m_n} - p_{\overline{A}}^{(+)m_n}\right) ; ,$$

To calculate $\mathcal{P}(n_1, n_2)$ one sums over all subsets $(\{i_j\},\{k_l\})$

$$\mathcal{P}(n_1, n_2) = \sum_{\substack{i_1<i_2<\cdots<i_{n_1} \\ k_1<k_2<\cdots<k_{n_2}}, i_j \neq k_l} \mathcal{P}\left(\{i_j\},\{k_l\}\right) \ , \qquad 1 \leq i_j, k_l, m_n \leq N$$

The upper limit on a fraction of primary A

Let ϵ_A be the fraction of A in $\mathcal{E}$

3. Let $\mathcal{P}(\epsilon)$ be the probability that the observed results are reproduced for a given ϵ_A. Hence

$$\mathcal{P}(\epsilon_A) = \sum_{n_1,n_2}^{n_1+n_2\leq N} \mathcal{P}(n_1, n_2)\epsilon_A^{n_1}\,(1-\epsilon_A)^{n_2}$$

4. Upper limit on ϵ_A at a given confidence level ξ:

$$P(\epsilon_A) \geq 1 - \xi$$

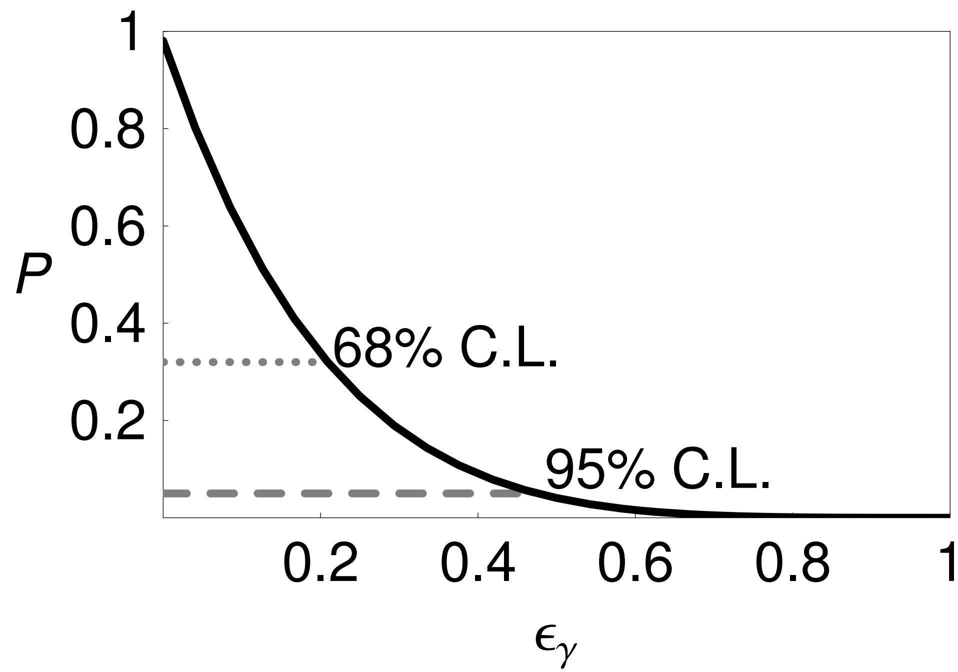
5. Correction because of cuts:

$$\lambda = \frac{m_{\mathrm{lost}}}{m}\,, \qquad \epsilon_{A,\,\mathrm{true}} = \frac{\epsilon_A}{1-\lambda+\lambda\epsilon_A}$$

Example

all AGASA events with known muon content
and energies $E_{\rm obs} > 8 \cdot 10^{19}$ eV: N=6

Event	$p_{\gamma}^{(+)}$	$p_{\bar{\gamma}}^{(+)}$
1	0.000	1.000
2	0.001	0.998
3	0.013	0.921
4	0.003	0.887
5	0.000	0.580
6	0.000	0.565



Result: $\epsilon_{\gamma,\rm true} < 0.50$ at 95% C.L.

AGASA+Yakutsk, $E > 10^{20}$ eV, muons: $\epsilon_{\gamma,\rm true} < 0.36$

Rubtsov et al, '06:

The most probable chemical composition

1. Input

set of N observed events with energies $\mathcal{E}$,

$p_{A_1}^{(+)j}$, $p_{A_2}^{(+)j}$, generally $p_{A_1}^{(+)j} + p_{A_2}^{(-)j} \neq 1$

Steps

2. Probability $\mathcal{P}(n_1, n_2)$: among N observed events, n_1 initiated by A_1 with $\mathcal{E}$ and n_2 initiated by A_2 with $\mathcal{E}$:

The probability to have i_1-th, . . . , i_{n_1}-th events ($i_1 < \cdots < i_{n_1}$) induced by A_1 with $\mathcal{E}$ and k_1-th, . . . , k_{n_2}-th events ($k_1 < \cdots < k_{n_2}$, $i_j \neq k_l$) induced by A_2 with $\mathcal{E}$:

$$\mathcal{P}\left(\{i_j\},\{k_l\}\right) = \prod_{i_j} p_{A_1}^{(+)i_j} \prod_{k_l} p_{A_2}^{(+)k_l} \prod_{m_n \neq i_j, k_l} \left(1 - p_{A_1}^{(+)m_n} - p_{A_2}^{(+)m_n}\right);,$$

To calculate $\mathcal{P}(n_1, n_2)$ one sums over all subsets $(\{i_j\},\{k_l\})$

$$\mathcal{P}(n_1, n_2) = \sum_{\substack{i_1 < i_2 < \cdots < i_{n_1} \\ k_1 < k_2 < \cdots < k_{n_2}}, i_j \neq k_l} \mathcal{P}\left(\{i_j\},\{k_l\}\right) \ , \qquad 1 \leq i_j, k_l, m_n \leq N$$

The most probable chemical composition

Let us suppose that $\epsilon_{A_{1,2}}$ are the fractions of $A_{1,2}$

3. Let $\mathcal{P}(\epsilon_{A_1})$ be the probability that the observed results are reproduced for a given set $(\epsilon_{A_1}, \epsilon_{A_2} = 1 - \epsilon_{A_1})$. Hence

$$\mathcal{P}(\epsilon_{A_1}) = \sum_{n_1, n_2}^{n_1+n_2 \leq N} \mathcal{P}(n_1, n_2) \epsilon_{A_1}^{n_1} \left(1 - \epsilon_{A_1}\right)^{n_2}$$

4.a The allowed ϵ_{A_1} at a given confidence level ξ:

$$P(\epsilon_{A_1}) \geq 1 - \xi$$

4.b The most probable composition: maximization of $\mathcal{P}(\epsilon_{A_1})$

5. Correction because of cuts:

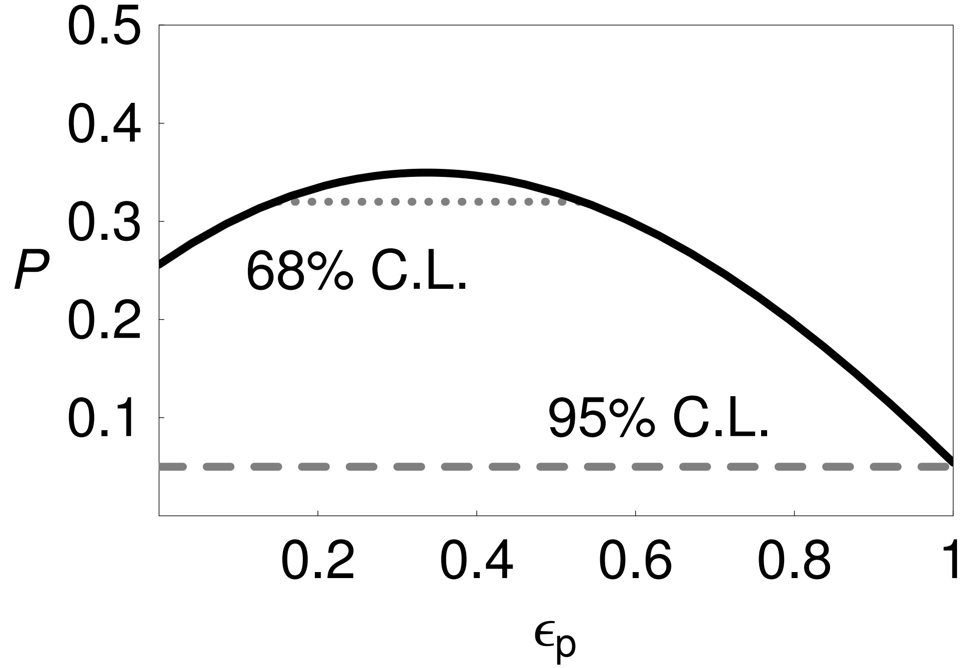
$$\lambda = \frac{m_{\text{lost}}}{m}, \qquad \epsilon_{A_1}^{\text{true}} = \frac{\epsilon_{A_1}(1 - \lambda_{A_2})}{1 - \lambda_{A_1} + \epsilon_{A_1}(\lambda_{A_1} - \lambda_{A_2})}.$$

Example

Two AGASA and two Yakutsk events with known muon content and energies $E_{\rm obs} > 1.5 \cdot 10^{20}$ eV:

N=4 , $A_1 = p$, $A_2 = Fe$

Event	$p_p^{(+)}$	$p_{\rm Fe}^{(+)}$
1	0.254	0.136
2	0.295	0.135
7	0.186	0.814
8	0.413	0.587



Result: most probable $\epsilon_p = 0.35$

$0.15 < \epsilon_p < 0.54$ at 68% C.L.

What is the primary of an observed event?

energy-related parameters of a shower — E-parameters
composition-related parameters of a shower — c-parameters

Both parameters are reconstructed with some errors

The probability distribution that the primary particle which produced an actual shower with the observed E-parameters equal to $E_{\rm obs}$ would rather produce a shower with these parameters equal to $E_{\rm rec}$:

$$g_E(E_{\rm rec}, E_{\rm obs})$$

The probability distribution that a shower with measured C-parameters equal to $\mathbf{c}$ could produce detector readings corresponding to $\mathbf{c}'$:

$$g_c(\mathbf{c}', \mathbf{c}).$$

Steps

1. for each primary one generates a library of simulated showers : the same direction, $E_s \sim E_{obs}$, e.g. $< 0.5E_{obs} < E_s < 2E_{obs}$
2. following the experimental procedure for each event one finds E_{rec}
3. one assigns to each simulated shower a weight $w_1 = g_E(E_{\rm obs}, E_{\rm rec})$
4. one assigns to each simulated shower an additional weight $w_2 = (E_s/E_{obs})^\alpha$ to mimic the real power-law spectrum

Output:
The distribution of the parameters $\mathbf{c}$ for the showers consistent with the real one by E-parameters is given by

$$f_A(\mathbf{c}) = \frac{1}{\mathcal{N}} \sum_i g_c(\mathbf{c}, \mathbf{c}_{iA}) w_{1,iA} w_{2,iA}$$

Results

If the event is unlikely being initiated by the primary A, one can estimate of the probability it could be initiated by the primary A:

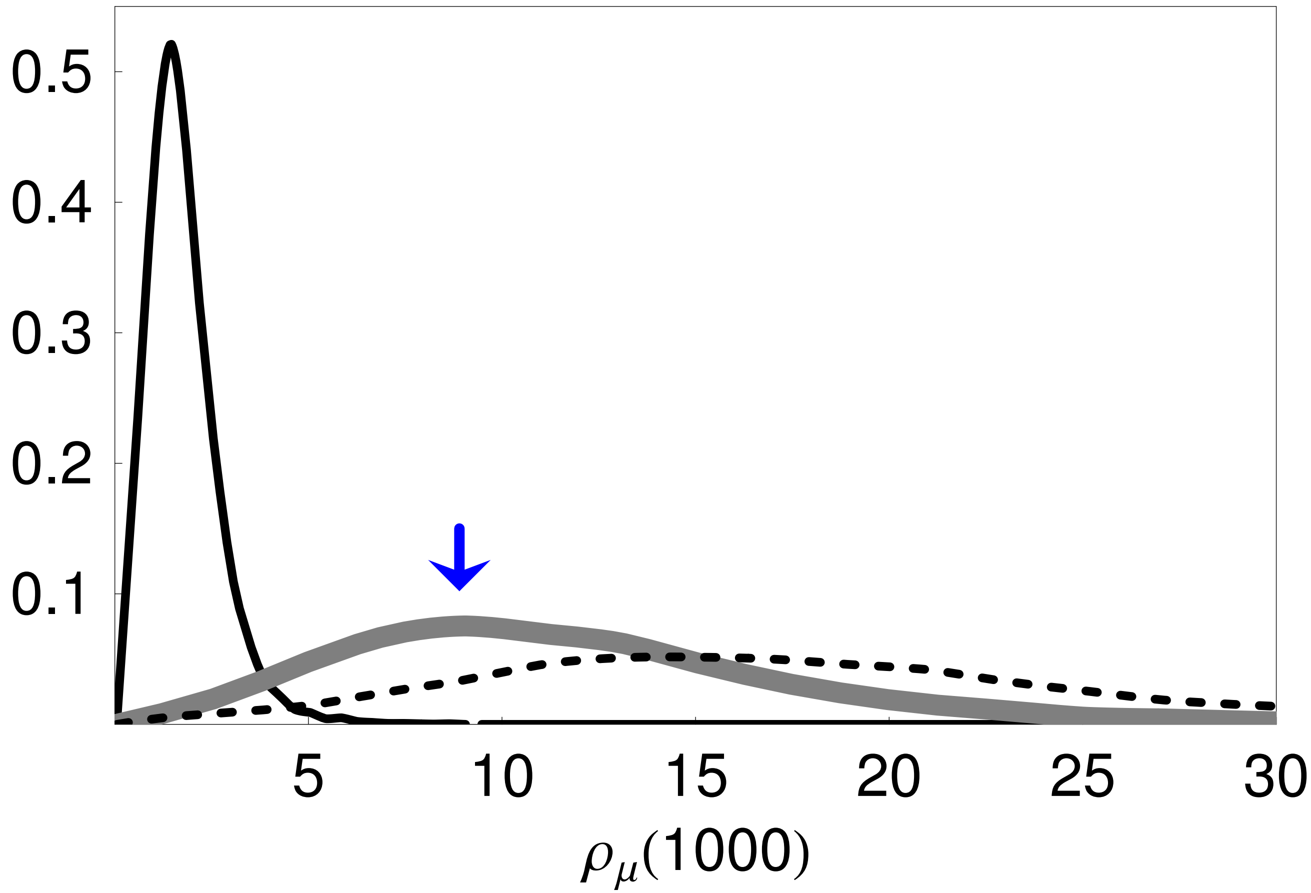
$$p_{A_1} = F_A(\mathbf{c}_{\text{obs}}) \equiv \int\limits_{f_A(\mathbf{c}) \le f_A(\mathbf{c}_{\text{obs}})} f_A(\mathbf{c}) d\mathbf{c}$$

one can test the hypotheisi that the primary was either A_1 or A_2. Then $p_{A_1} + p_{A_2} = 1$ and

$$p_{A_k} = \frac{f_{A_k}(\mathbf{c}_{\text{obs}})}{f_{A_1}(\mathbf{c}_{\text{obs}}) + f_{A_2}(\mathbf{c}_{\text{obs}})}$$

Example

The highest energy AGASA event $2.46 \cdot 10^{20}$ eV



Distributions of muon densities f_A of simulated events: thin dark line, $A = \gamma$; thick gray line, $A = p$; dashed line, $A =$Fe.

What else?

- Many types of $\mathbf{c}$-parameters
- Many types of primaries
- Unknown primary
- Only $\mathcal{E}$ depends on energy systematics