Bose Stars

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Markov readings, 13 May 2023, Moscow

Outline

- Axion Bose Star collapse

  D.Levkov, A.Panin, & IT, PRL 118 (2017) 011301

- ♥ Formation of Bose Stars

  D.Levkov, A.Panin, & IT, PRL 121 (2018) 151301

- Radio-emission of Axion Stars

  D.Levkov, A.Panin, & IT, Phys.Rev.D 102 (2020) 023501

- Instability of rotating Bose stars

  A. Dmitriev, D.Levkov, A.Panin, & IT, Phys.Rev.D 104 (2021) 023504

- ♥ Growth of Bose Stars

  A. Dmitriev, D.Levkov, A.Panin, & IT, arXiv: 2305.01005

- ♥ Destructon of Bose Stars

  P. Tinyakov, IT, & K. Zioutas, JCAP 01 (2016) 035

- Bose star is a self-gravitating clump of Bosons in the lowest energy state.

Ruffini & Bonazzola, Phys. Rev. 187 (1969) 1767

- May appear in Dark Matter models with light Bose particles. Mainstream candidates - QCD axion or ALP in general:

Axion stars

IT, Sov. Astron. Lett. 12 (1986) 305

- Vast literature, but little attention to the problem of their formation.

Bose-condensation

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- Interactions are needed to form Bose condensate
- But ALP couplings are extremely small

QCD axions

- Solve strong CP problem
- CDM: $m \approx 26\ \mu$eV
- $\lambda \sim 10^{-50}$

String axions

- Appear in string models
- Fuzzy DM: $m \sim 10^{-22}$ eV
- $\lambda \sim 10^{-100}$

- Relaxation time is enhanced due to large phase space density f

IT, Phys. Lett. B **261** *(1991) 289*

$$\boxed{\tau_R^{-1} \sim \sigma\, v\, n\, (1 + f)} \qquad \text{where } f \sim \frac{n}{(mv)^3} \gg 1$$

which is still not enough to beat small λ (except in rare axion miniclusters)

Bose condensation by gravitational interactions

D.Levkov, A.Panin, & IT, PRL 121 (2018) 151301

Are we crazy?

- No
- $f \gg 1$ — classical fields
- $v \ll 1$ — nonrelativistic approximation
- Gravity but no other interactions

$$\psi(t, x)$$
$$U(t, x)$$

Field equations for light DM (Scrödinger-Poisson system)

$$i\partial_t \psi = -\Delta\psi/2m + mU\psi$$
$$\Delta U = 4\pi G(\underbrace{m|\psi|^2}_{\rho} - \langle\rho\rangle)$$

Bose star is a stationary solution:
$\psi = \psi_s(r)\mathrm{e}^{-i\omega t}$

Solving these equations in time, we find Bose condensation!

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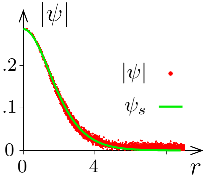

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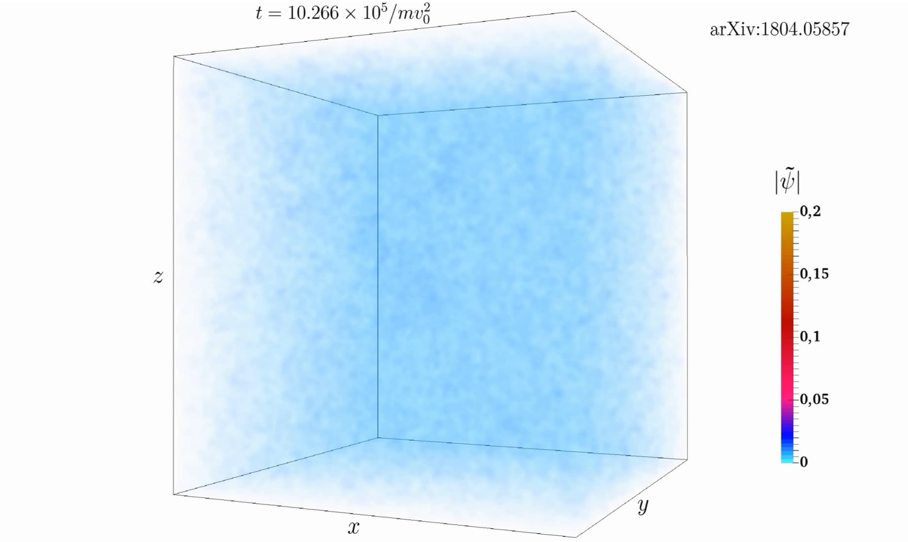
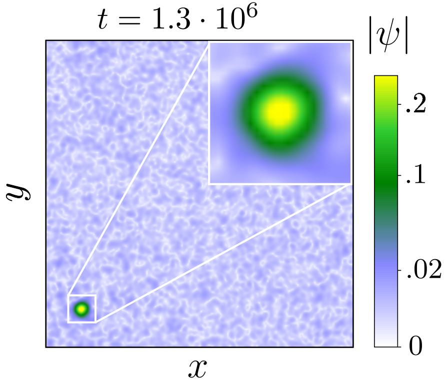
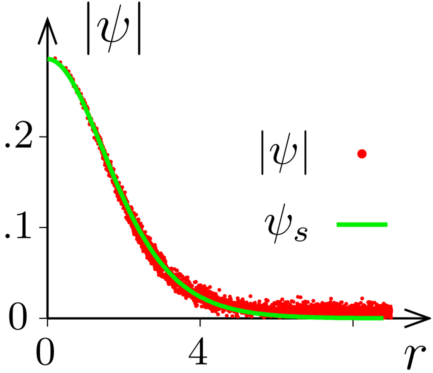
Time evolution



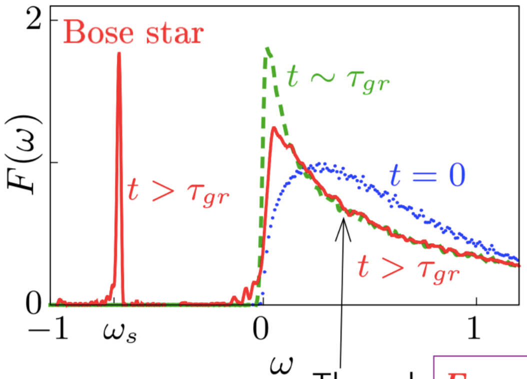




We observe formation of a Bose star at $t = \tau_{gr}$

Energy distribution at different moments of time

$$F(\omega,\, t) \equiv \frac{dn}{d\omega} = \int d^3x \int \frac{dt_1}{2\pi}\, \psi^*(t,x)\psi(t+t_1,x)\, \mathrm{e}^{i\omega t_1 - t_1^2/\tau_1^2}$$



Thermal: $F \propto \omega^{-1/2}$

Landau equation — derivation

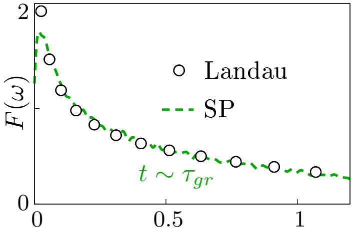
- Perturbative solution of Schrödinger-Poisson equation
- Kinetic approximations $(mv)^{-1} \ll x, \; (mv^2)^{-1} \ll t$
- Compute Wigner distribution

$$f_p(t,x) = \int d^3y \, \mathrm{e}^{-ipy} \langle \psi(x+y/2)\psi^*(x-y/2) \rangle$$

random phase average

$$\partial_t f_p + \frac{p}{m}\nabla_x f_p - m\nabla_x \bar{U} \nabla_p f_p = \mathrm{St}\; f_p$$

D.Levkov, A.Panin, & IT, PRL 121 (2018) 151301



Good agreement of lattice F and kinetic f,

$$F_\omega = \frac{mpf_p}{2\pi^2}, \qquad \omega = \frac{p^2}{2m}$$

We solve kinetic equation in the form

$$\partial_t F_\omega = \mathrm{St}\; F_\omega$$

Kinetics

Landau equation — derivation

- Perturbative solution of Schrödinger-Poisson equation
- Kinetic approximations $(mv)^{-1} \ll x, \; (mv^2)^{-1} \ll t$
- Compute Wigner distribution

$$f_p(t,x) = \int d^3y \, \mathrm{e}^{-ipy} \langle \psi(x+y/2)\psi^*(x-y/2) \rangle$$

random phase average

$$\partial_t f_p + \frac{p}{m}\nabla_x f_p - m\nabla_x \bar{U} \nabla_p f_p = \mathrm{St}\, f_p \equiv \boxed{\frac{f}{\tau_R}}$$

$\leftarrow$ relaxation time

$\Uparrow$

$f_p^3 \leftarrow$ Bose amplification

Time to Bose star formation: $\tau_{gr} = b\,\tau_R = \dfrac{4\sqrt{2}b}{\sigma_{gr} v\, n f}$

$\uparrow$

$O(1)$ correction

Time to Bose star formation

$$\tau_{gr} = \frac{4\sqrt{2}b}{\sigma_{gr} v n f}$$

Rutherford cross section: $\sigma_{gr} \approx 8\pi(mG)^2\Lambda/v^4$

$$\Lambda = \log(mvR)$$

Coulomb logarithm

Average phase-space density: $f = 6\pi^2 n/(mv)^3$

$$\tau_{gr} = \frac{b\sqrt{2}}{12\pi^3}\frac{mv^6}{G^2\Lambda n^2}$$

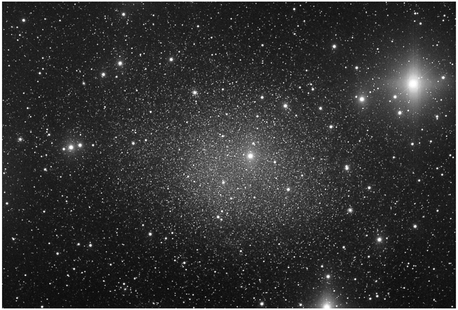
- Strongly depends on local quantities: $n,\ v,\ f$
- Involves global logarithm $\Lambda = \log(mvR)$

Models: fuzzy dark matter

String axions

$$\tau_{bs} \sim 10^6\,\text{yr} \left(\frac{m}{10^{-22}\,\text{eV}}\right)^3 \left(\frac{v}{30\,\text{km/s}}\right)^6 \left(\frac{0.1\,M_\odot/\text{pc}^3}{\rho}\right)^2$$

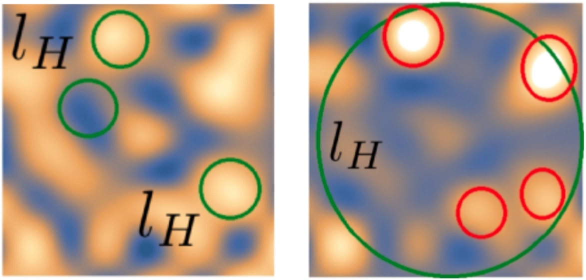
Fornax dwarf galaxy



$$v \sim 11\ \text{km/s}$$
$$\rho \sim 0.1\,M_\odot/\text{pc}^3$$
$$\tau_{bs} \sim 1000\ \text{yr}$$

Universe filled with Bose stars!

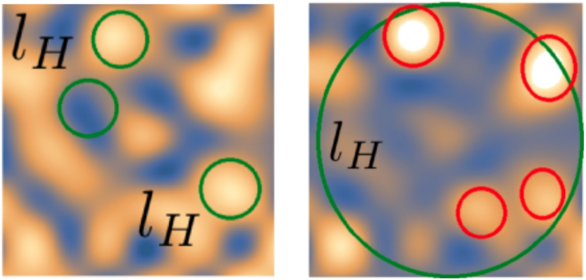
PQ phase transition after inflation → Miniclusters



- After phase transition $0 < \theta < 2\pi$ from horizon to horizon, but $\theta \approx \mathrm{const}$ on a horizon scale l_H
- Peculiar initial amplitude of oscillations when m_a turns on
- Dark Matter should be very clumpy

QCD axion cosmology

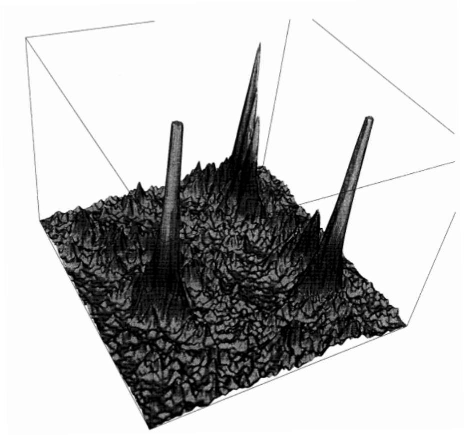
PQ phase transition after inflation → Miniclusters

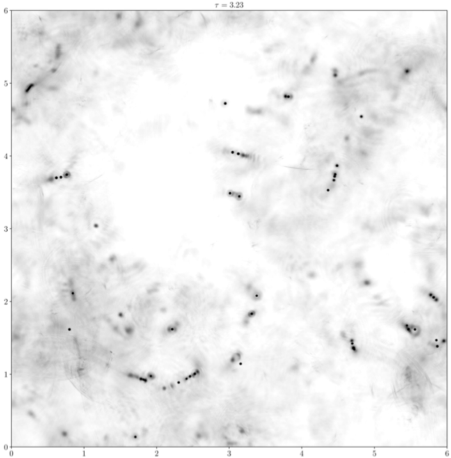


- Mass scale of the clumps is set by $M \sim 10^{-11}\, M_\odot$, which is DM mass within horizon at $T_{\rm osc} \approx 1$ GeV
- Naively, initial DM density contrast is $\delta\rho_a/\rho_a \equiv \Phi \sim 1$
- In fact, very dense objects can form, $\Phi \gg 1$, since for $\theta \sim 1$ the axion attractive self-coupling is non-negligible,

$$V(a) = m^2 f_a^2 \left(\frac{\theta^2}{2} - \frac{\theta^4}{4!} + \dots \right)$$

Minicluster seeds formation at QCD

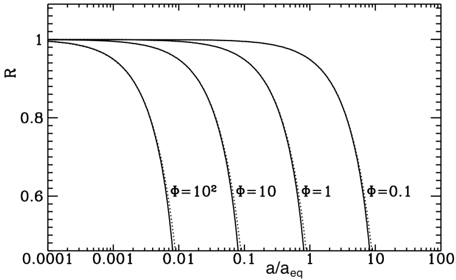




The height of the plot is cut at $\Phi = 20$.
E.Kolb & IT, Phys.Rev. D49 (1994) 5040

A.Vaquero, J.Redondo, J.Stadler
arXiv:1809.09241

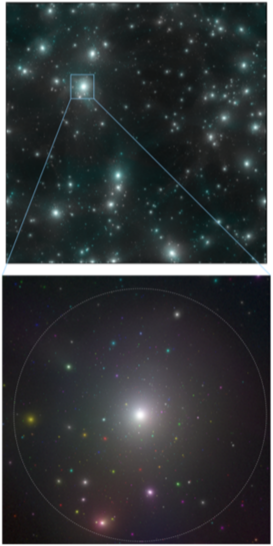
Minicluster formation around equality



A clump becomes gravitationally bound at $T \approx \Phi\, T_{eq}$, i.e. its density today

$$\rho_{mc} \approx 140\Phi^3(1+\Phi)\bar{\rho}_a(T_{eq})$$

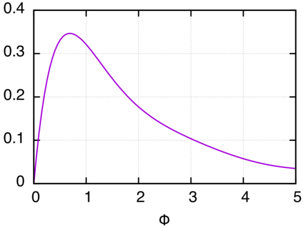
E.Kolb & IT, Phys.Rev. D50 (1994) 769



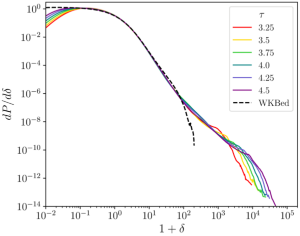
B.Eggemeier, et al arXiv:1911.09417

Bose-star formation: QCD axions

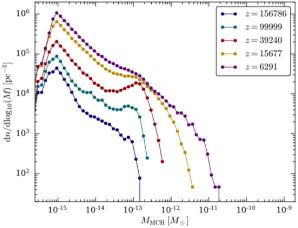
$$\tau_{bs} \sim \frac{10^9\ \text{yr}}{\Phi^4}\left(\frac{M_c}{10^{-13}\ M_\odot}\right)^2\left(\frac{m}{26\ \mu\text{eV}}\right)^3$$



Mass fraction in miniclusters
E.Kolb & IT (1994)



A.Vaquero et al (2019)



B.Eggemeier et al (2019)

- $\Phi \sim 1 \Rightarrow \tau_{bs} \sim 10^9$ yr
- $\Phi \sim 10^3 \Rightarrow \tau_{bs} \sim$ hr

Universe filled with axion Bose stars!

Bose-star growth

- The problem of the growth of a Bose star seems to be very difficult.
  - It is intrinsically inhomogeneous, involves gravity, etc.
- Attempts to approach it in the past were based on calculations of the cross-section for particle capture by the star, see e.g.

J. Chan, S. Sibiryakov, and W. Xue, 2207.04057

- But the solution turned out to be simple and analytical,

A. Dmitriev, D. Levkov, A. Panin, and I.I Tkachev, arXiv:2305.01005.

  - The new paradigm:
    - Condensation = flux of particles from the "bath" into the ground state.
    - Kinetics in the bath is the key.
    - The star eats everything she was given.
  - A miracle that allows for analytics — evolution is self-similar.

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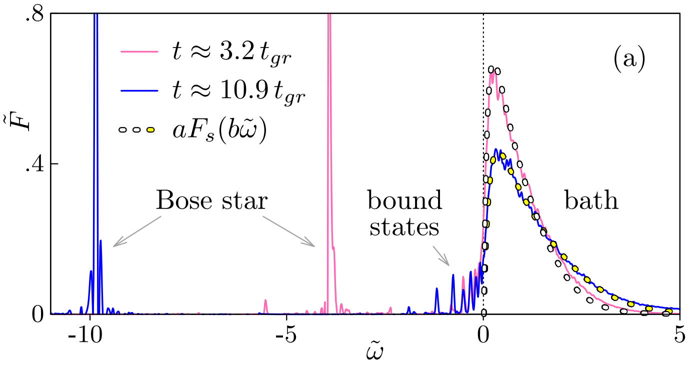
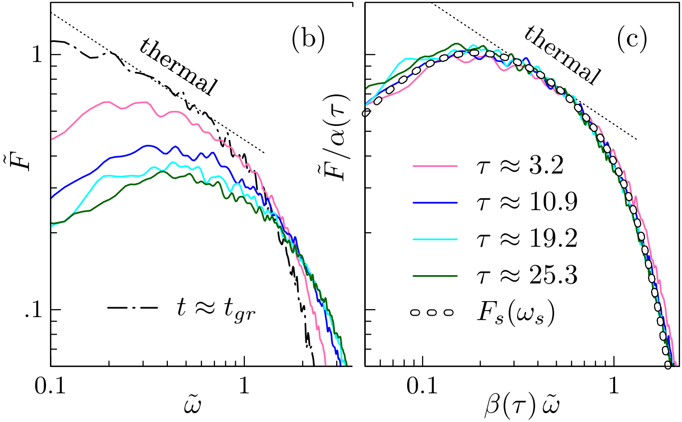


Particle distribution function $F(\omega)$ at two moments of time.
Chain points – solution of the Landau kinetic equation.

$M_* \equiv M_{bs} + M_e = \int_{\omega<0} F\, d\omega$, while M_b is the same integral over $\omega > 0$,

$$M = M_{bs} + M_e + M_b.$$

Bose-star growth



Distribution functions $F(\omega)$ before (left) and after self-similar transformation (right).

$$\tilde{F}(t, \tilde{\omega}) = \alpha F_s(\beta\tilde{\omega})\,, \quad \alpha = \tau^{-1/D}\,, \quad \beta = \tau^{2/D-1}\,,$$

$\tau = t/t_{gr}$, chain points, F_s – solution of the transformed Landau kinetic equation.

Bose-star growth

Self-similarity gives time-dependent mass $M_b \propto \tau^{k_M}$ and energy $E_b \propto \tau^{k_E}$ with

$$k_M = 1 - 3/D\,, \quad k_E = 2 - 5/D\,, \quad 3k_E - 5k_M = 1 \ .$$

D is determined by the boundary conditions, which change in our case.

Define

$$k_M(\tau) \equiv \frac{d\ln M_b}{d\ln\tau} \quad \text{and} \quad k_E(\tau) \equiv \frac{d\ln E_b}{d\ln\tau}\,.$$

They satisfy the self-similar law, $3k_E - 5k_M \approx 1$, if change slowly:

$$dk_{M,E}/d\ln\tau \ll 1.$$

Then, the conservation laws $M_b = M - M_*$ and $E_b = E - E_*$ give

$$d\ln\tau \approx 3d\ln(E - E_*) - 5d\ln(M - M_*)\,,$$

or

$$(1 - E_*/E)^3(1 - M_*/M)^{-5} \approx (\tau - \tau_i)/\tau_*\,.$$

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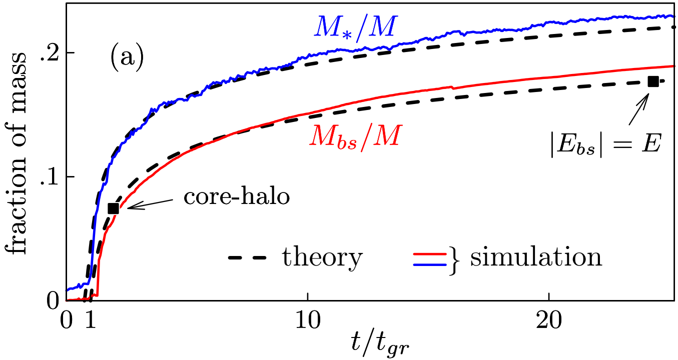
In E_* we can count only the ground state $E_* \approx -\gamma M_{bs}^3$, with $\gamma \approx 0.0542\, m^2 G^2$,

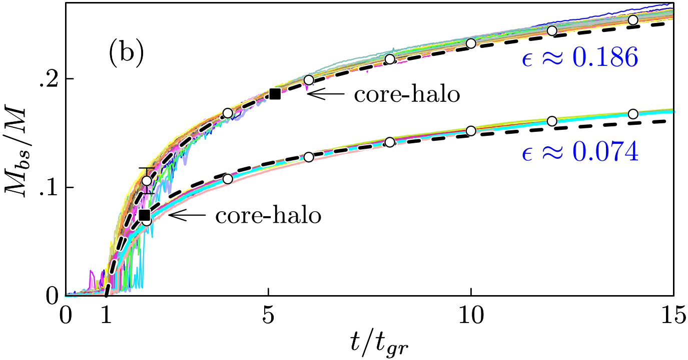
$$(1 + x_{bs}^3/\epsilon^2)^3 (1 - x_e - x_{bs})^{-5} \approx \tau\,,$$

where $\epsilon^2 \equiv E/\gamma M^3$ and $(x_{bs} + x_e) = M_*(\tau)/M$.



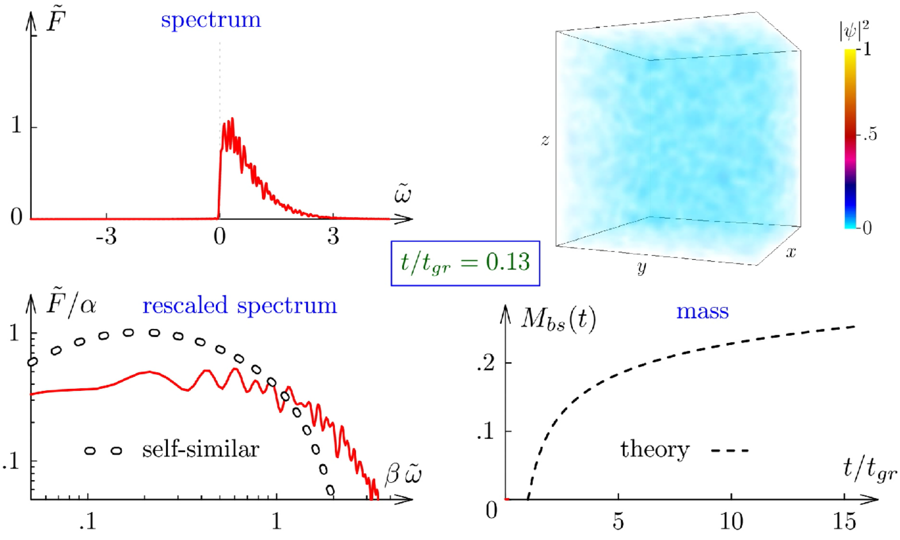
Bose star mass at $\epsilon = 0.074$, dashed lines – theory.

Bose-star growth



Evolutions of $M_{bs}(t)$ in 11+22 simulations at $\epsilon = 0.074$ and 0.186. Circles – averages over simulations with given ϵ, dashed lines – theory.

Bose-star growth



Phenomenological implications (QCD axions)

- Less diffuse DM -> smaller signals in DM detectorts
- But rare strong signals during encounters with debris of tidally disrupted miniclusters

  P.Tinyakov, IT and K. Zioutas, JCAP 1601 (2016) 035
- Gravitational microlensing and femtolensing

  E.Kolb & IT, Astrophys.J 460 (1996) L25

  M.Fairbairn, et. al, PRL 119 (2017) 021101
- Decay of Bose stars
  - Decay to relativistic self
    - Resolution of tension between low and high z observations?

      Z.Berezhiani, A.Dolgov & IT, Phys.Rev. D 92 (2015) 061303
  - Decay to radiophotons
    - Relation to FRB?

      IT, JETP Letters 101 (2015) 1

      A.Iwazaki, PRD 91(2015) 023008
- Specific signal in Gravitational waves

Minicluster abundance

Typical miniclusters with $\mathbf{\Phi \approx 1}$:

- $\mathbf{10^{25}}$ in the Galaxy
- $\mathbf{10^{10}\ pc^{-3}}$ in the Solar neighborhood
- Minicluster radius $\sim \mathbf{10^{7}\ km}$
- Direct encounter with the Earth once in $\mathbf{10^{5}\ years}$
- During encounter density increases by a factor $\mathbf{10^{8}}$ for about a day

But, some miniclusters are destroyed in encounters with stars. This may change the prospects for DM detection.

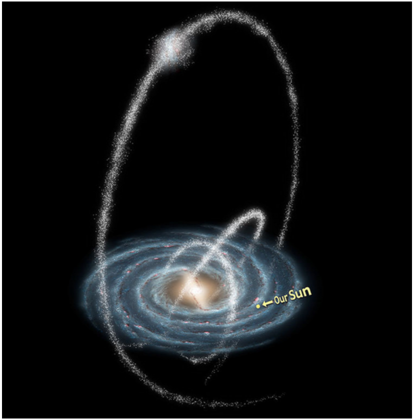
Tidal streams from miniclusters

Probability of a minicluster disruption

$$P(\Phi) = 0.022 \left(\frac{n}{100}\right) \Phi^{-3/2} (1+\Phi)^{-1/2}$$

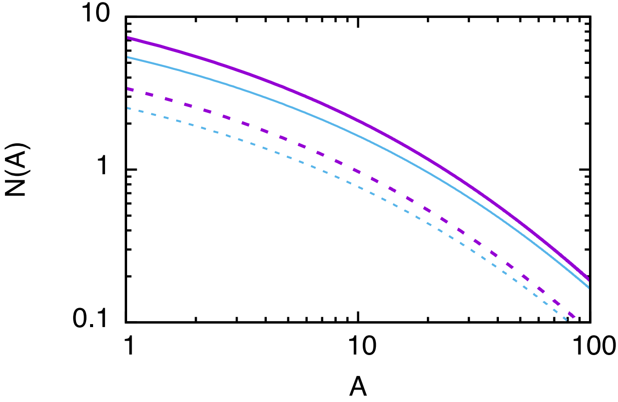
Just disk crossings. No actual orbits integration.

P.Tinyakov, IT and K. Zioutas, JCAP 1601 (2016) 035



Crossing tidal streams from miniclusters

Mean number of encounters with axion streams producing amplification factor larger than A, as a function of A. Twenty year observation interval is assumed.



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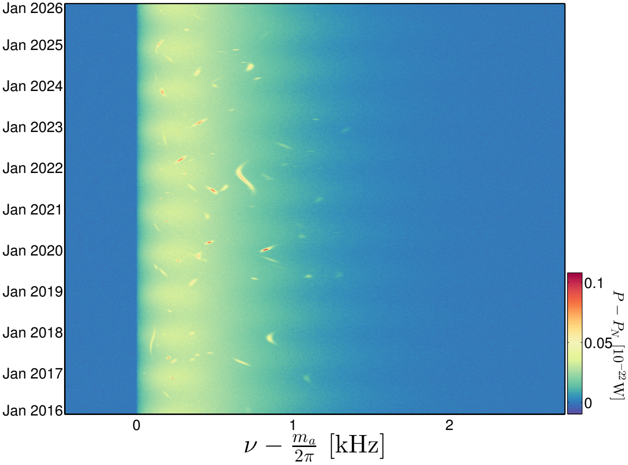
Crossing tidal streams from miniclusters

Simulation of expected PWS in cavity experiments



C. O'Hare and A. Green, Phys.Rev. D95 (2017) 063017

Conclusions

- Bose condensation by gravitational interactions is very efficient
- Large fraction of axion dark matter may consist of Bose stars
- Phenomenological implications of Bose star existence are reach and deserve further studies